

**DYNAMICAL MODELS OF WINDS FROM ROTATING
HOT STARS**

by

Steven Robert Cranmer

A dissertation submitted to the Faculty of the University of Delaware in
partial fulfillment of the requirements for the degree of Doctor of Philosophy in
Physics

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by

Steven Robert Cranmer

Approved: _____

Henry R. Glyde, Ph.D.

Chairman of the Department of Physics and Astronomy

Approved: _____

John C. Cavanaugh, Ph.D.

Interim Associate Provost for Graduate Studies

I certify that I have read this dissertation and that in my opinion it meets the academic and professional standard required by the University as a dissertation for the degree of Doctor of Philosophy.

Signed: _____
Stanley P. Owocki, Ph.D.
Professor in charge of dissertation

I certify that I have read this dissertation and that in my opinion it meets the academic and professional standard required by the University as a dissertation for the degree of Doctor of Philosophy.

Signed: _____
James MacDonald, Ph.D.
Member of dissertation committee

I certify that I have read this dissertation and that in my opinion it meets the academic and professional standard required by the University as a dissertation for the degree of Doctor of Philosophy.

Signed: _____
Derck Massa, Ph.D.
Member of dissertation committee

I certify that I have read this dissertation and that in my opinion it meets the academic and professional standard required by the University as a dissertation for the degree of Doctor of Philosophy.

Signed: _____
David Seckel, Ph.D.
Member of dissertation committee

I certify that I have read this dissertation and that in my opinion it meets the academic and professional standard required by the University as a dissertation for the degree of Doctor of Philosophy.

Signed: _____
Gary Zank, Ph.D.
Member of dissertation committee

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ABSTRACT

The hottest and most massive stars (spectral types O, B, Wolf-Rayet) have strong stellar winds that are believed to be driven by line scattering of the star’s continuum radiation field. The atmospheres and winds of many hot stars exhibit the effects of rapid rotation, pulsation, and possibly surface magnetic fields, inferred from observations of ultraviolet spectral lines and polarization. The complex time variability in these observations is not yet well understood. The purpose of this dissertation is to model the dynamics of winds around rotating hot stars and synthesize theoretical observational diagnostics to compare with actual data.

Before dealing with rotation, however, we derive the theory of radiative driving of stellar winds, and uncover several new useful aspects of the theory for spherical, nonrotating stars. The presence of *limb darkening* of the stellar radiation is found to be able to increase the mass flux \dot{M} by 10–15% over standard models assuming a uniformly-bright star, and the wind’s asymptotic terminal velocity v_∞ should decrease by the same amount. We also introduce a new approximation method for estimating the terminal velocity, which is both conceptually simpler and more physically transparent than existing approximation algorithms. Finally, from theoretical line profile modeling we find that observational determinations of v_∞ may be underestimated by several hundred km s^{-1} if *unsaturated* P Cygni lines are used.

Rotation affects a star by introducing centrifugal and Coriolis forces, decreasing the effective gravity and making the star oblate. This in turn redistributes the emerging radiative flux to preferentially heat the stellar poles, an effect known as *gravity darkening*. Although previous models have computed the increase in equatorial mass flux due to the lower effective gravity there, none have incorporated gravity darkening. We find that the brighter (darker) flux from the poles (equator) has a much stronger impact on the mass flux, increasing (decreasing) the mass loss and local wind density. This, in addition to the existence of *nonradial* radiation forces from a rotating star, which tend to point latitudinally away from the equator and azimuthally opposite the rotation, produces a net *poleward* deflection of wind streamlines. This is contrary to the “wind compressed disk” model of Bjorkman

and Cassinelli, and also seems incompatible with observational inferences of equatorial density enhancements in some systems. This work is ongoing, and we are endeavoring to include all the relevant physics in hydrodynamical simulations.

We also dynamically model spectral-line *time variability* by inducing corotating nonaxisymmetric structure in the equatorial plane of a hot-star wind. By varying the radiation force over localized “star spots,” the wind develops fast and slow streams which collide to form corotating interaction regions (CIRs) similar to those in the solar wind. We synthesize P Cygni type line profiles for a stationary observer, and find that “discrete absorption components” (DACs) accelerate slowly through the profiles as complex nonlinear structures rotate in front of the star. We also examine the photospheric origin of such variability, in a preliminary manner, by deriving the theory of stellar pulsations, waves, and discontinuities. Although most observed low-order pulsation modes are evanescently damped in the photosphere, we find that the presence of an accelerating wind can allow waves of *all* frequencies to propagate radially. We thus make a first attempt at outlining the possible “photospheric connection” between interior and wind variability that observations are beginning to confirm.