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A Simple Approach to Mutual Irradiation Heating in Close Binary Systems

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1. Star-Star Heating

The process by which the radiation from one component of a binary system affects the photosphere of its companion is often called the "reflection effect." However, for early-type systems with radiative envelopes, this is a misnomer, since most of the incident radiation is *absorbed.* The resulting temperature rise due to this irradiation is the primary physical diagnostic of this effect, and it is what this brief note attempts to predict analytically.

Consider a binary system, with spherical stars (radii R_1 and R_2, luminosities L_1 and L_2), their centers separated by a distance D. Let us deal first with the radiation from star 2 incident upon a specified point on star 1, and further only examine the "line-of-centers" point on star 1 which is closest to star 2. The effect of mutual irradiation will be strongest there. Note, of course, that the stars themselves actually *iterate* this effect between one another, since, e.g., when star 2 heats up star 1, that in turn affects the illumination back onto star 2, and so on. This, fundamentally, is a multiple scattering problem.

The simple approach we adopt (for each "iteration") is to see how mutual irradiation affects a grey atmosphere computed using the Eddington approximation. Ignoring differences in the spectral energy distributions between the two stars, we can simply invoke radiative equilibrium to scale up the effective temperature of star 1 by including the incident flux from star 2,

$$\sigma T_{\rm eff}^4 = \sigma T_{\rm eff}^4(\text{star 1}) + \mathcal{F}_{\rm incident} \quad (1)$$

$$= \frac{L_1}{4\pi R_1^2} + \frac{A_2 L_2}{4\pi (D-R_1)^2} \ , \quad (2)$$

where A_2 is a bolometric reflection albedo, which is unity for radiative envelopes. However, this effective albedo has been shown to drop to $A_2 \approx 0.5$ for convective envelopes.

The optical depth dependence of the temperature, $T(\tau)$, has been derived by Anderson & Shu (1977) to take the above "plane parallel" irradiation into account, but also the effects of the geometry of the system. If the sky of the illuminated star is partially filled by the illuminating star, some of the incident flux will enter at *grazing* angles, preferentially heating the upper photosphere ($\tau \ll 1$). In the limit of a completely filled sky, the atmosphere will heat up more and more, since photons have nowhere to eventually "escape." Anderson & Shu (1977) assume a grey temperature law of the form

$$\sigma T^4(\tau) = \tfrac{3}{4}\sigma T_{\rm eff}^4 (\tau + Q_0) \ , \tag{3}$$

where Q_0 is a constant to be determined. The resulting limb-darkening solution for the emergent specific intensity $I(\mu, \tau = 0)$ can be used to re-compute the flux, and the only remaining unknown is Q_0.

Anderson & Shu (1977) also assume that, in thermal equilibrium, the flux emerging from star 1 must be eventually radiated into the solid angle subtended by the *sky*, Ω_S, which is smaller than the total hemispheric value of 2π. This idealized thermal equilibrium demands that

$$F_{\rm tot} = \int_{\Omega_S} I(\mu)\mu \, d\Omega = \frac{3F_{\rm tot}\,\Omega_S}{4\pi} \left(Q_0 \langle \mu \rangle + \langle \mu^2 \rangle \right) \ , \tag{4}$$

where the average denoted by angle brackets is defined by

$$\langle f(\mu) \rangle \equiv \frac{\int_{\Omega_S} f(\mu)\, d\Omega}{\int_{\Omega_S} d\Omega} = \frac{2\pi}{\Omega_S} \int_{\mu=0}^{\mu_e} f(\mu)\, d\mu \ . \tag{5}$$

The last integral assumes axial symmetry about the line of centers, and defines the edge angle

$$\mu_e = \cos\theta_e = \sqrt{1 - \left(\frac{R_2}{D - R_1} \right)^2} \ . \tag{6}$$

Also, the sky solid angle is given by $\Omega_S = 2\pi\mu_e$. Working out the terms in eq. (4) results in

$$Q_0 = \frac{2}{3} \left(\frac{2 - \mu_e^3}{\mu_e^2} \right) \ . \tag{7}$$

Thus, when $\mu_e \rightarrow 1$ (i.e., no companion star), $Q_0 \rightarrow 2/3$, which is the classical Eddington value. However, for $\mu_e < 1$, Q_0 grows rapidly, which indeed preferentially heats up the outer layers of the photosphere. (Note, of course, that Anderson & Shu's assumption of "thermal equilibrium" here denotes the *end result* of multiple iterative heating, and not just a single iteration.)

Finally, then, the temperature distribution is

$$T(\tau) = \left\{ \frac{3}{4\sigma} \left[\frac{L_1}{4\pi R_1^2} + \frac{A_2 L_2}{4\pi (D - R_1)^2} \right] (\tau + Q_0) \right\}^{1/4} \ . \tag{8}$$

Let us write the *ratio* of this enhanced temperature to the unaffected, single-star temperature, at a fiducial optical depth, say $\tau = 2/3$. Note that this ratio isolates the irradiation heating effect from other effects, such as gravity darkening. Thus,

$$\frac{T}{T_0}(\tau = 2/3) = \left\{ \left[1 + A_2 \left(\frac{L_2}{L_1} \right) \left(\frac{R_1}{D - R_1} \right)^2 \right] \left(\frac{2 + \mu_e^2 - \mu_e^3}{2\mu_e^2} \right) \right\}^{1/4} , \qquad (9)$$

which depends only on the ratios (L_2/L_1), (R_1/D), and (R_2/D), as well as the assumed albedo A_2. If we compare this result to that of Cranmer (1991, 1993), who computed the mutual, fully-iterated, and self-consistent geometrical irradiation heating in close binaries, we find that the above relation significantly *overestimates* the heating at the line-of-centers point, by factors of 1.5 to 6 for various systems. It is the Q_0-factor, not the overall enhancement in $T_{\rm eff}$, which seems to be the root of these discrepancies.

Actually, of course the re-radiation for a single iteration will enter the complete hemisphere ($\Omega_S \approx 2\pi$) above the surface of star 1. (It is only after many iterations that that star "knows" that the effective sky is diminished!) If this is the case, we can use Anderson & Shu's (1977) "unbiased" averages of $\langle\mu\rangle = 1/2$ and $\langle\mu^2\rangle = 1/3$ in eq. (4), and obtain their result,

$$Q_0 = \frac{2}{3} \left(\frac{2 - \mu_e}{\mu_e} \right) , \qquad (10)$$

which is consistently *smaller* than the exact value of Q_0 derived above (eq. [7]) for $\mu_e < 1$. Heating derived with this simpler value of Q_0 comes much closer to the full-geometry results of Cranmer (1991, 1993), and, e.g.,

$$\frac{T}{T_0}(\tau = 2/3) = \left\{ \left[1 + A_2 \left(\frac{L_2}{L_1} \right) \left(\frac{R_1}{D - R_1} \right)^2 \right] \left(\frac{1}{\mu_e} \right) \right\}^{1/4} \qquad (11)$$

is an adequate estimate of "reflection-effect" heating.

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