ASTR-6000 Seminar
COLLAGE: Coronal Heating, Solar Wind, & Space Weather

February 24, 2022

Solar wind acceleration mechanisms (yet another whirlwind tour)

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Dr. Thomas E. Berger

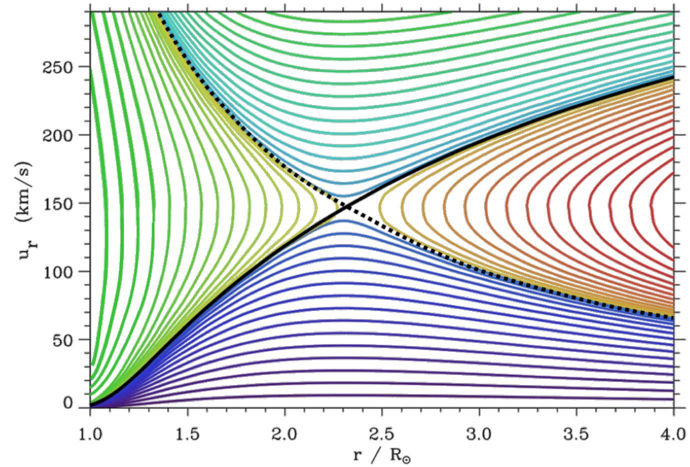
Outline

1. Generalizing the equation of motion, to get ready to look at…
2. `handson_solarwind_v1.ipynb`
3. Including physical processes that:
   - go beyond what Parker (1958) took into account, and
   - may be important to accelerating the solar wind
     a. Multi-fluid effects ($T_p \neq T_e \neq T_{ion}$)
     b. Non-Maxwellian particle distributions ($T_{\parallel} \neq T_{\perp}$)
     c. "Superradial" magnetic flux-tube expansion
     d. Alfvén wave pressure gradients

(1) *General thoughts*

- Parker's original isothermal model has a RHS that is negative near the Sun, zero at the critical point, and positive above it…

$$\left(u - \frac{c_i^2}{u}\right)\frac{du}{dr} = \left(\frac{2c_i^2}{r} - \frac{GM_\odot}{r^2}\right) = \text{RHS}$$



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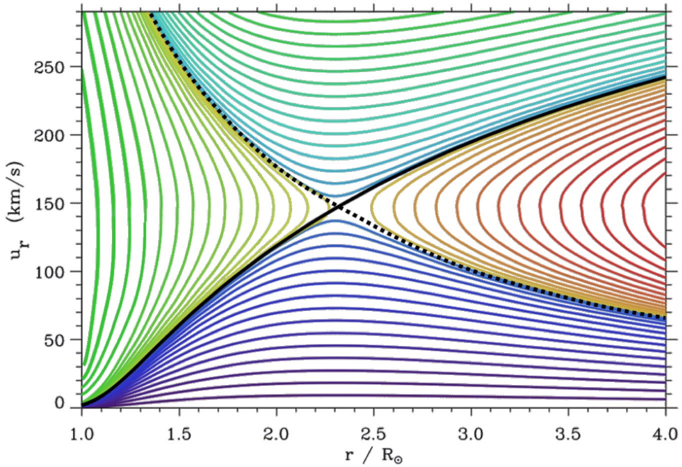

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- For a non-constant $T(r)$, there's one extra term in the gas-pressure gradient:

$$\text{RHS} = \frac{2c_i^2}{r} - \frac{dc_i^2}{dr} - \frac{GM_\odot}{r^2}$$

but it's usually not as large in magnitude as the $2c_i^2/r$ term.

- Note that, because $c_i(r)$ is no longer a constant, determining the **critical velocity** (LHS) depends on first determining the critical radius.

(1) *What to use for T(r) ?*

- In week 2, we derived an "RTV-lite" temperature law for a loop with $Q_{\text{heat}} = Q_{\text{cond}}$:

$$T = T_{\max}\left[1-\left(\frac{z}{L}-1\right)^{2}\right]^{2/7}$$

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- Cranmer & Schiff (2021) derived a cousin of that law for an **open-field solar-wind** region with a heating rate dropping like a power-law with radius:

$$Q_{\rm heat} \propto \left(\frac{R_\odot}{r}\right)^{\psi} \quad \Rightarrow \quad T = T_{\rm max}\left\{\frac{x_*^{\psi-2}}{\psi-3}\left[\frac{R_\odot}{r} - \left(\frac{R_\odot}{r}\right)^{\psi-2}\right]\right\}^{\delta}$$

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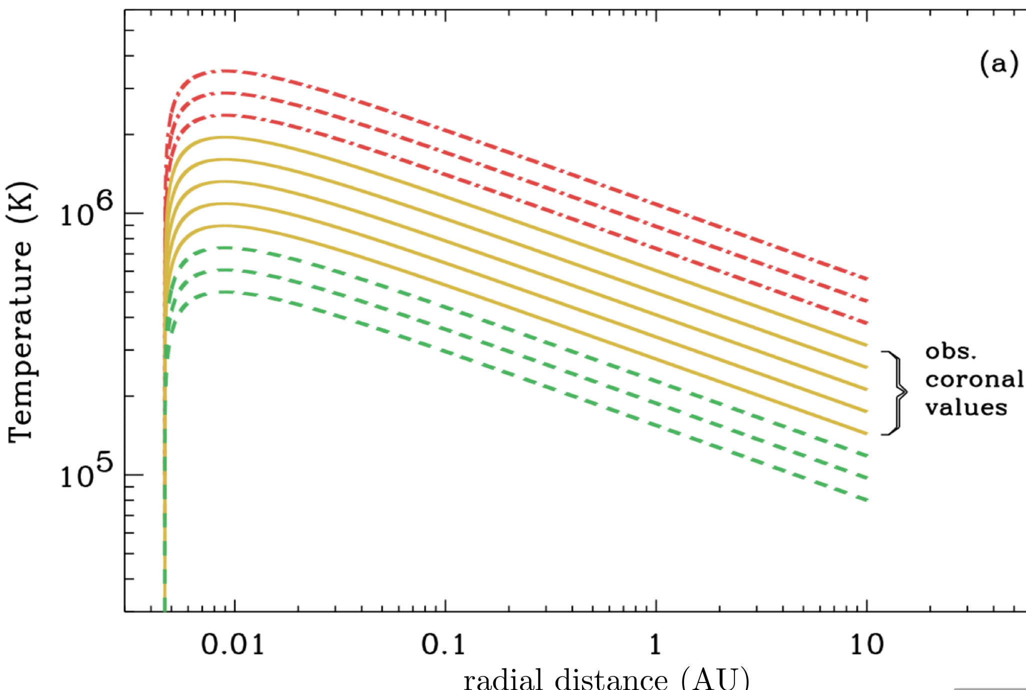

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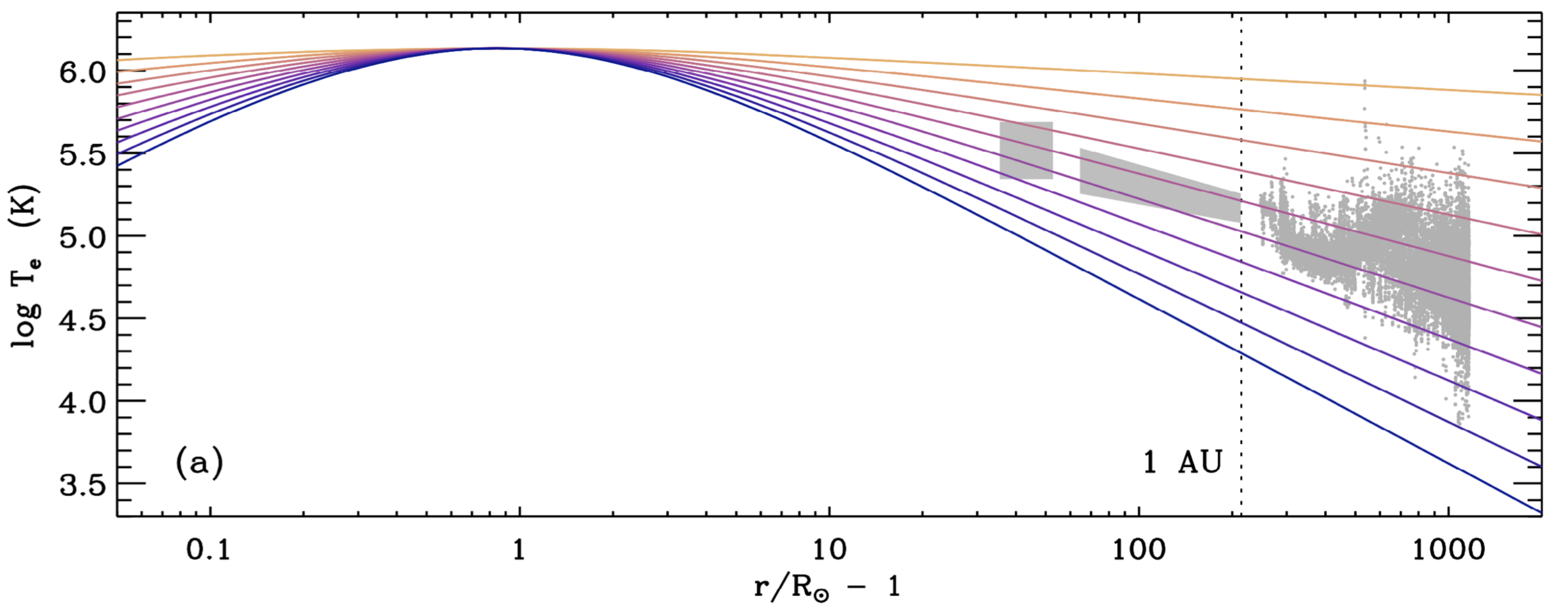
- For Spitzer-Härm conduction, $\delta = 2/7$, like above, but it can be varied, too…



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- The location of the critical point corresponds to the place where RHS = 0…

$$\text{RHS} \;=\; \frac{2c_i^2}{r} - \frac{dc_i^2}{dr} - \frac{GM_\odot}{r^2} \;+\; \text{other terms?}$$

- However, what if the function RHS(r) crosses zero multiple times? It can happen!

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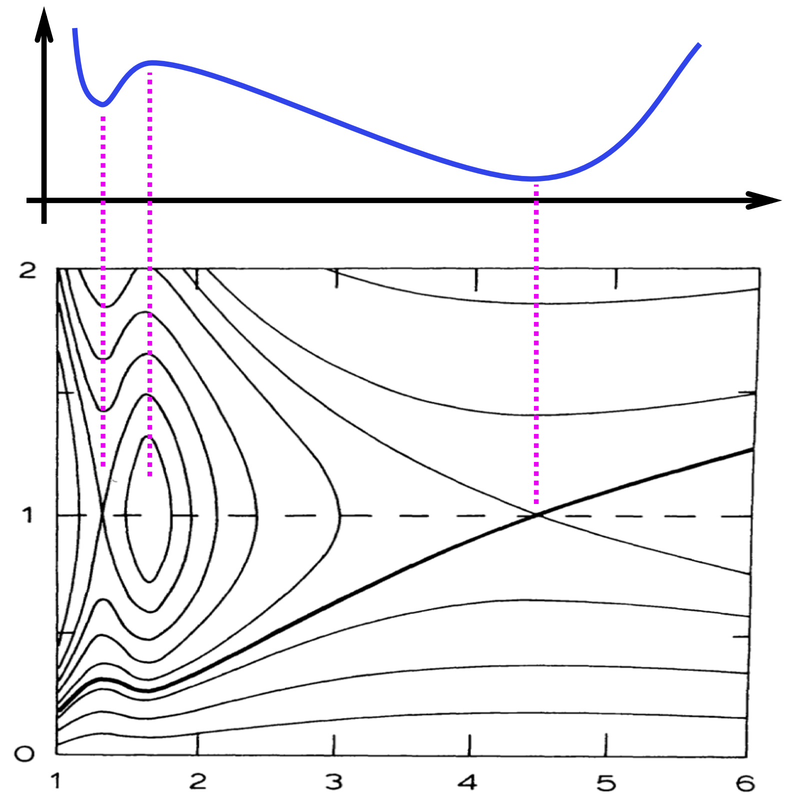
- However, what if the function RHS(r) crosses zero multiple times? It can happen!

- Kopp & Holzer (1976) found that these zeros correspond to places where the **integral of RHS** is either a minimum or maximum.

$$\text{RHSINT} = \int_{R_\odot}^{r} dr' \ \text{RHS}(r')$$

- The true critical point, which allows for transonic solutions that extend from the Sun to infinity, occurs at the point where RHSINT reaches its **global minimum.**



(2) *Let's look through the notebook*

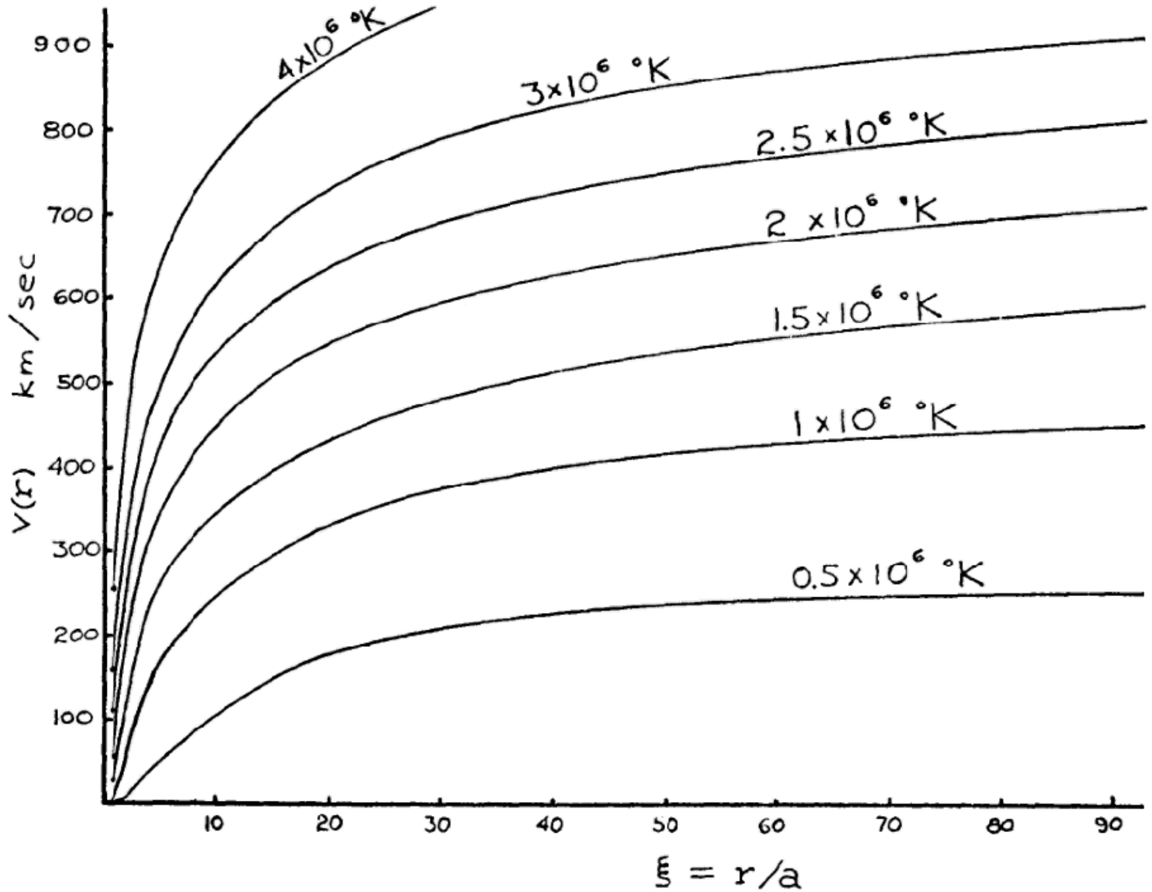


FIG. 1 —Spherically symmetric hydrodynamic expansion velocity $v(r)$ of an isothermal solar corona with temperature T_0 plotted as a function of r/a, where a is the radius of the corona and has been taken to be 10^{11} cm

(3a) *Multi-fluid effects*

- Each component of the plasma contributes to the total pressure gradient:

$$P_{\rm gas} = \sum_s n_s k_{\rm B} T_s \qquad\qquad s = p\,,\ e\,,\ \alpha\,,\ \&\ \text{minor/heavy ions}$$

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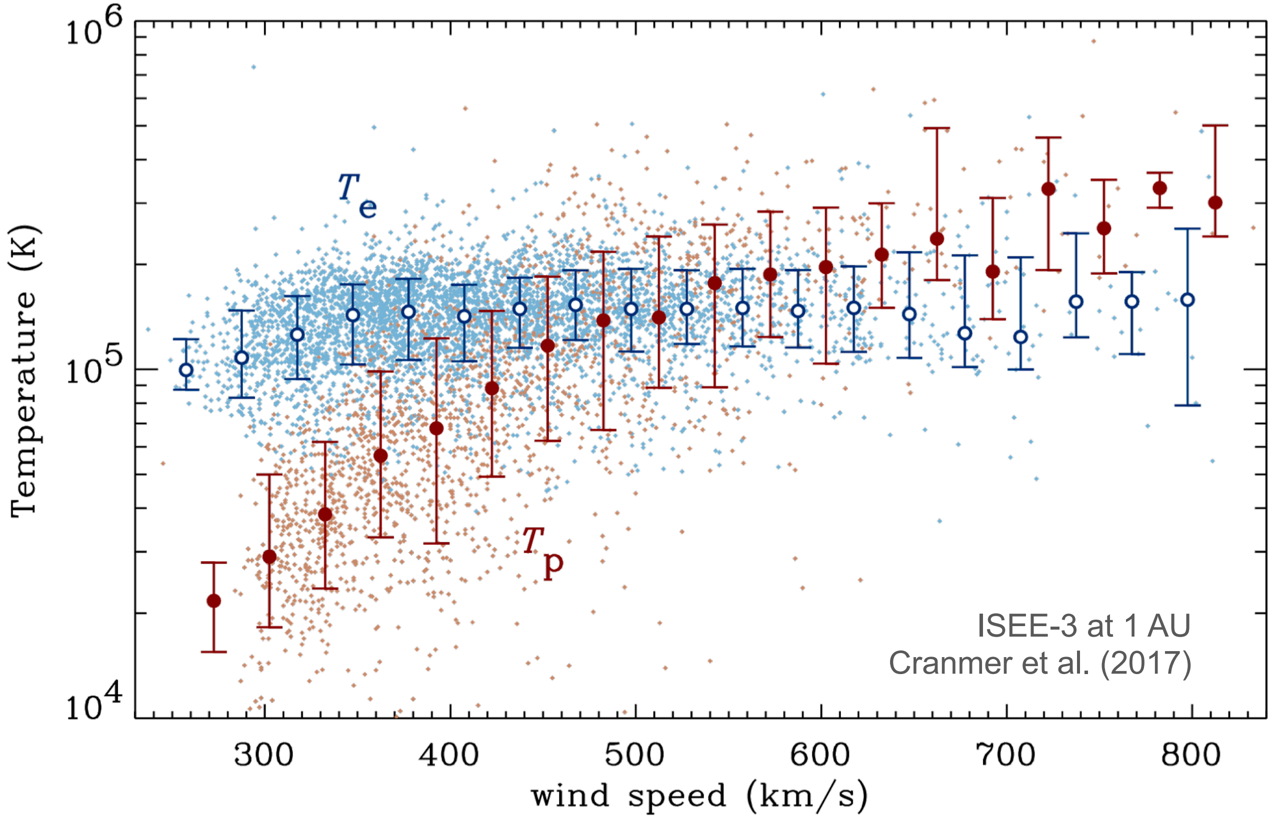
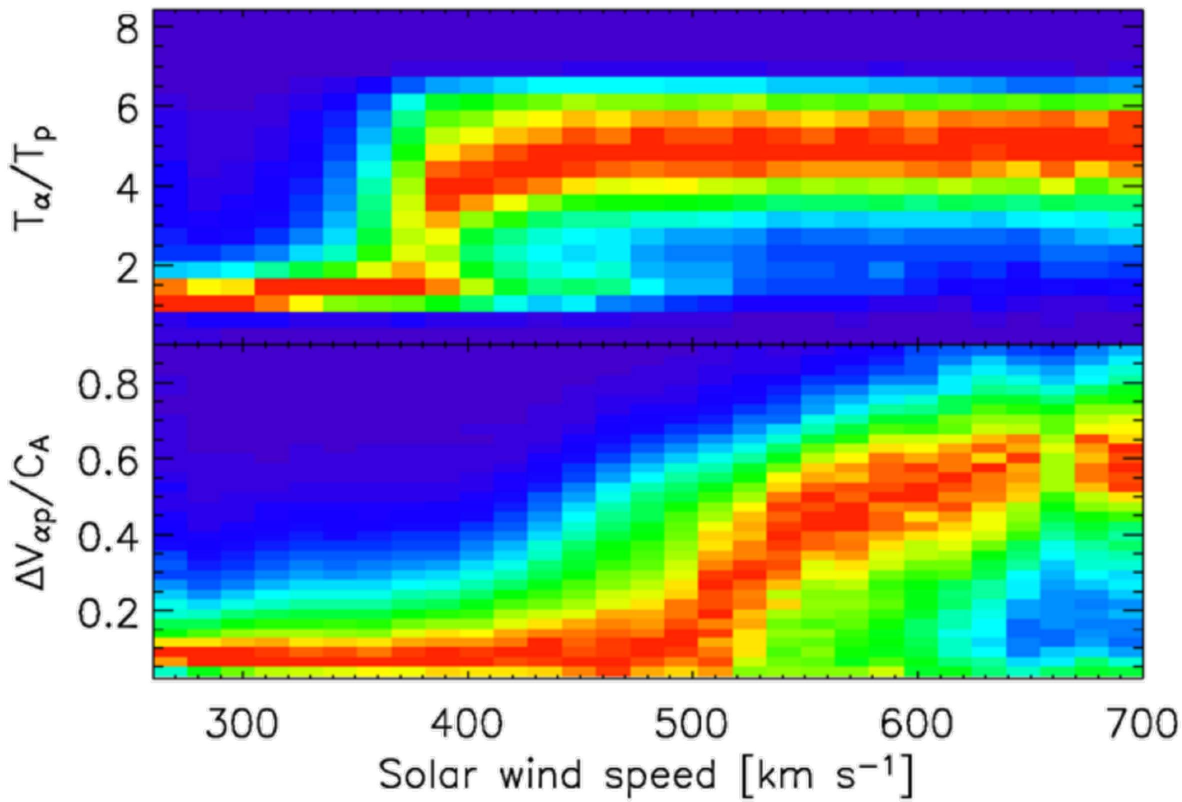
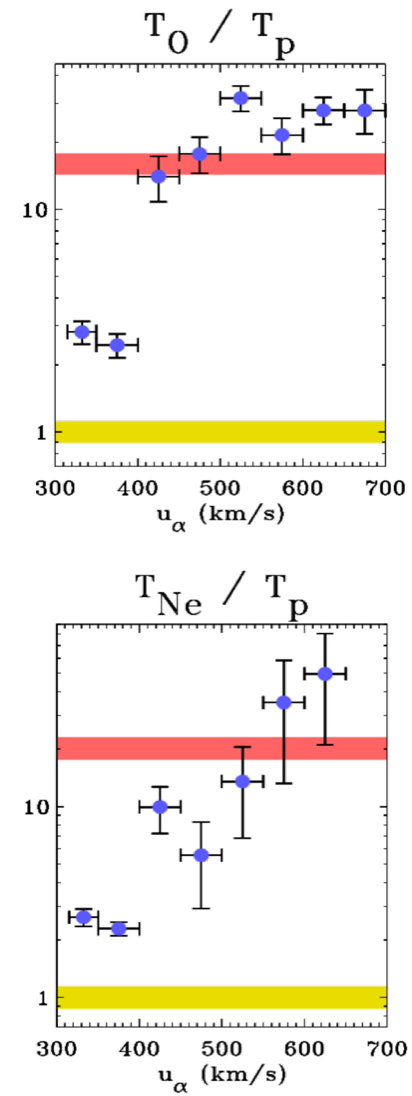
- Hydrogen (protons & electrons) is the most important ingredient…



(3a) *Multi-fluid effects*

- In the fast wind, helium & minor ions are often ***hotter*** & ***flow faster than*** protons.
- Many theoretical models emphasize "mass-proportional heating" (i.e., $T_{ion}/T_p = m_{ion}/m_p$) but the data often show T exceeding that…

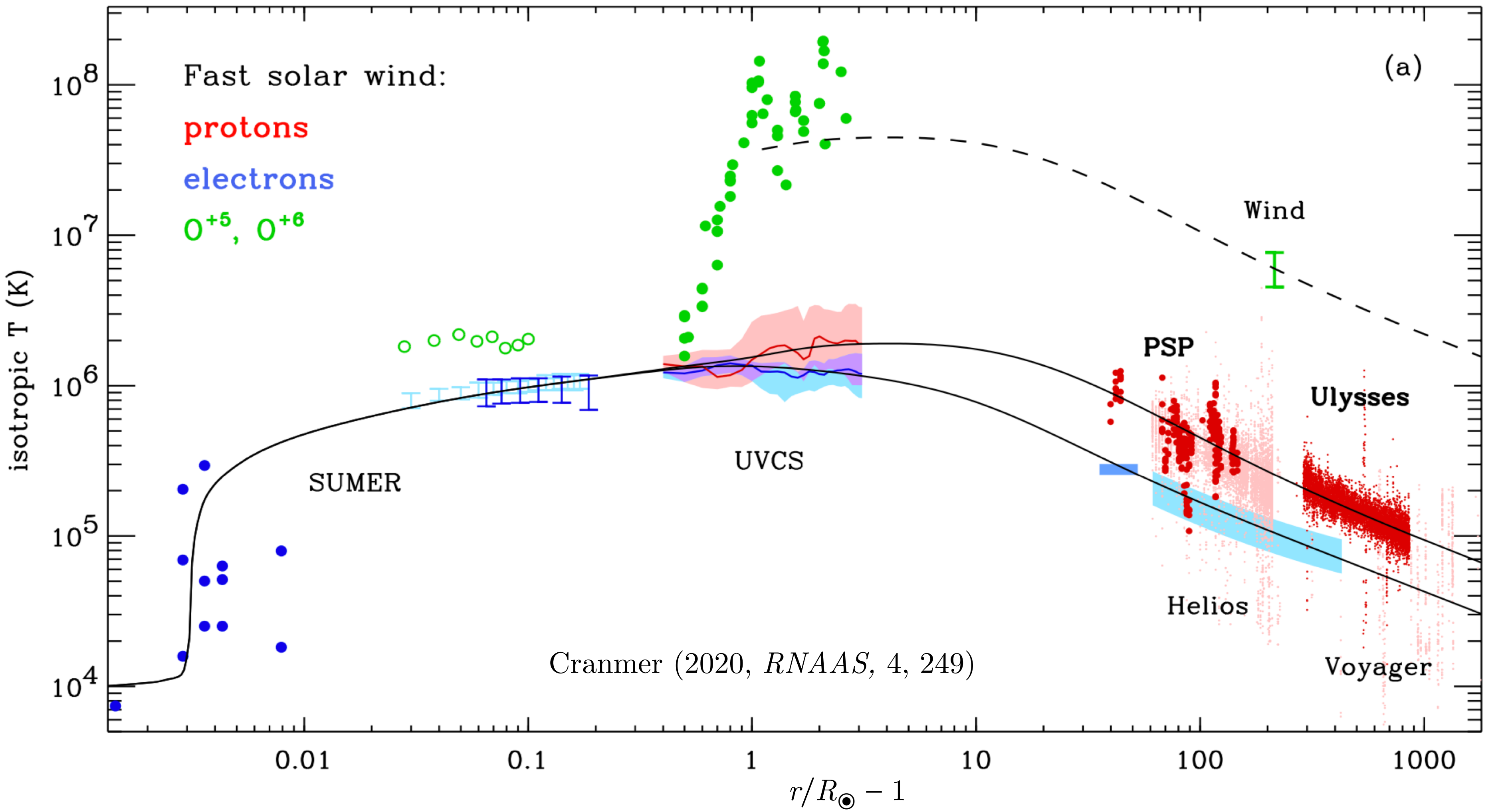




↑ Kasper et al. (2016) data from Collier et al. (1996) →

(3a) *Multi-fluid effects*

- Some of these departures from thermal equilibrium get their start in the corona…



(3a) *Multi-fluid effects*

- What to use in the solar wind equations? Ignoring the contribution of heavy/minor ions,

$$\rho \approx m_p n_p + m_\alpha n_\alpha$$

$$n_e \approx n_p + 2n_\alpha$$

$$P \approx n_p k_\mathrm{B} T_p + n_e k_\mathrm{B} T_e + n_\alpha k_\mathrm{B} T_\alpha$$

$$P = \rho c_i^2 \quad \text{where} \quad c_i^2 = \frac{k_\mathrm{B}\left[T_p + (1+2h)T_e + hT_\alpha\right]}{m_p(1+4h)} \quad \text{and} \quad h = \frac{n_\alpha}{n_p}$$

(3b) *Non-Maxwellian distributions*

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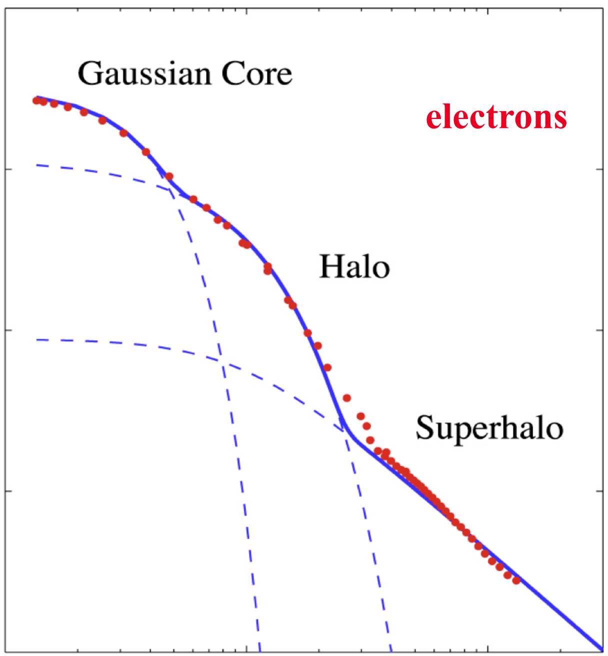
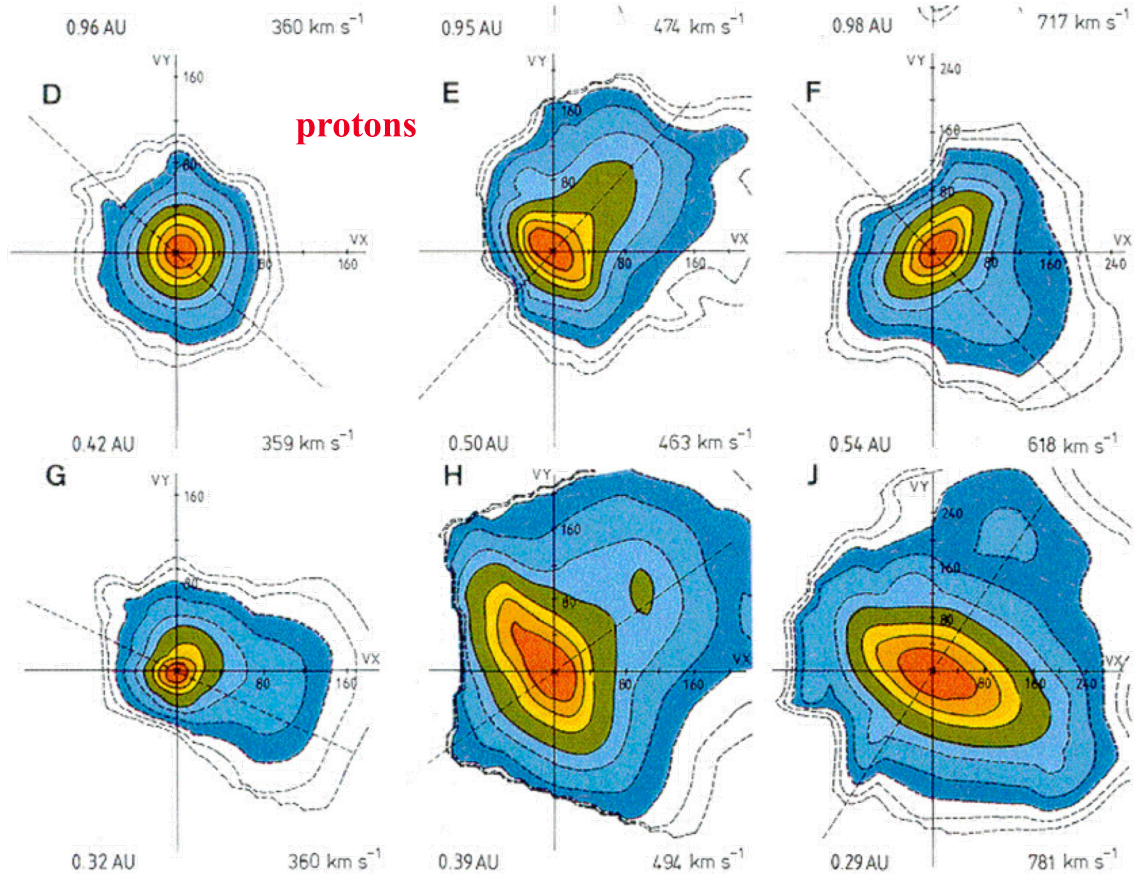
- Departures from equilibrium Maxwell-Boltzmann distributions are seen frequently in space plasmas… e.g., **anisotropies** (with respect to the magnetic field direction):

- One also sees departures as a function of particle **energy** (speed):



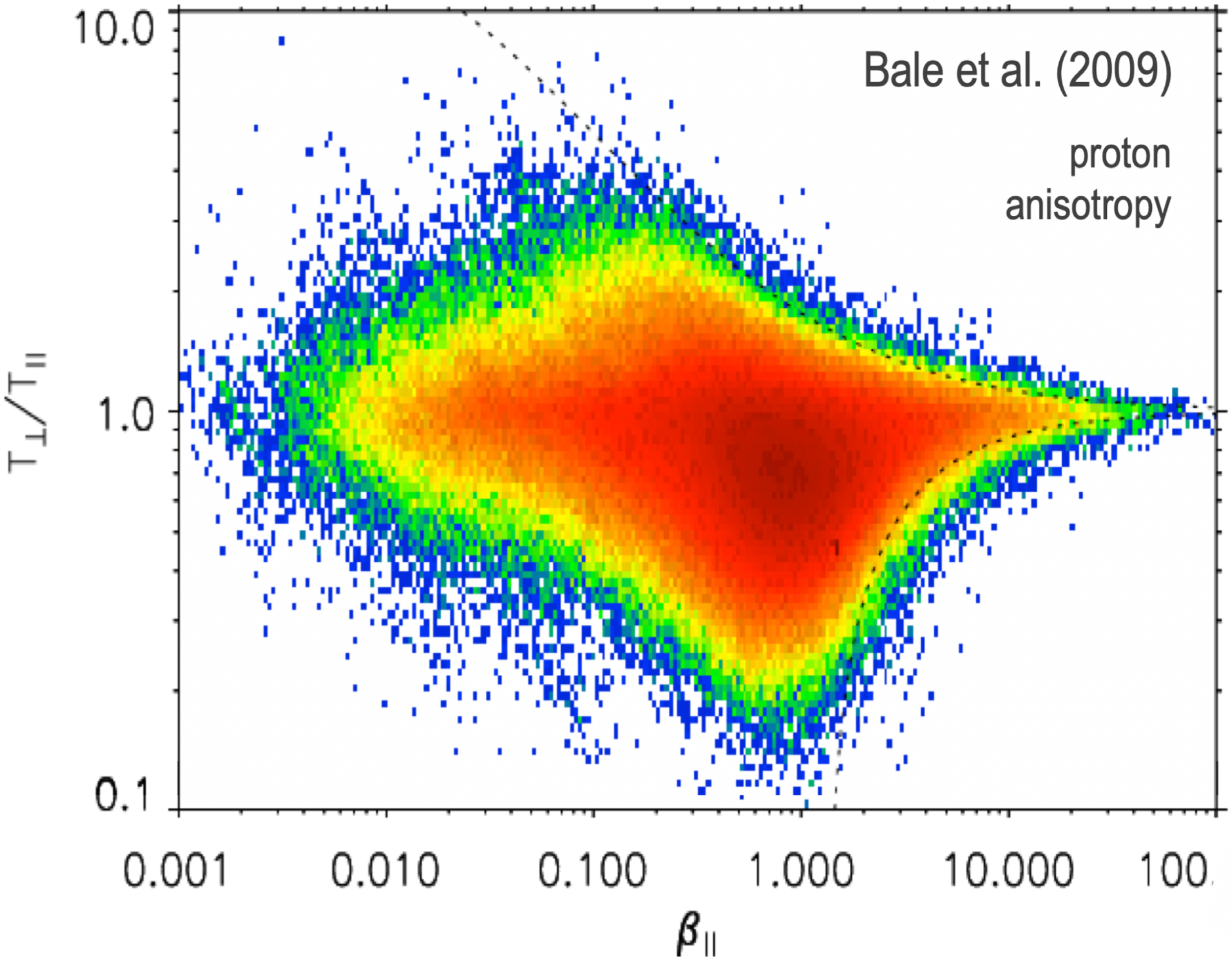
Kim, Yoon, Choe, & Wang (2015)



Marsch et al. (1982)

(3b) *Non-Maxwellian distributions*

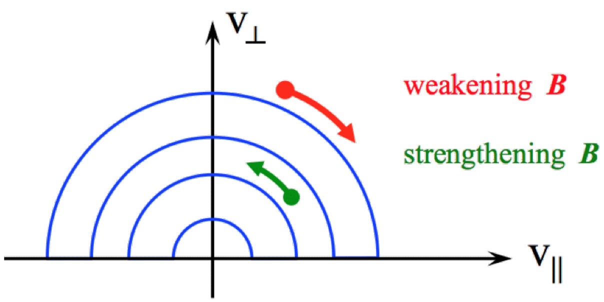
- Temperature anisotropies are characterized by fitting the distribution as "ellipsoidal," with unequal values of temperature parallel vs. perpendicular to the background **B**-field:

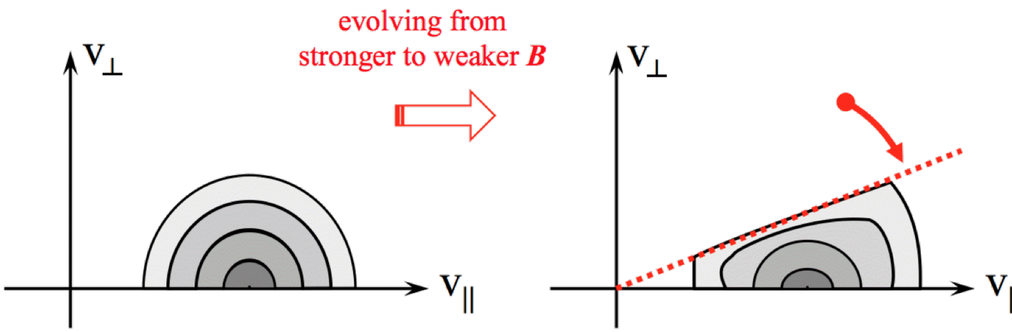


Patterns in proton anisotropy ratios (seen at 1 AU) point to plasma-scale instability limits and constraints on their coronal "history."

(3b) *Non-Maxwellian distributions*

- How do anisotropies arise?
- The solar wind expands through an ever-weakening magnetic field. If the plasma's magnetic moment was conserved, this would lead to particle scattering to higher values of $T_{||}$ and lower values of $T_{\perp}$





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- How do anisotropies arise?
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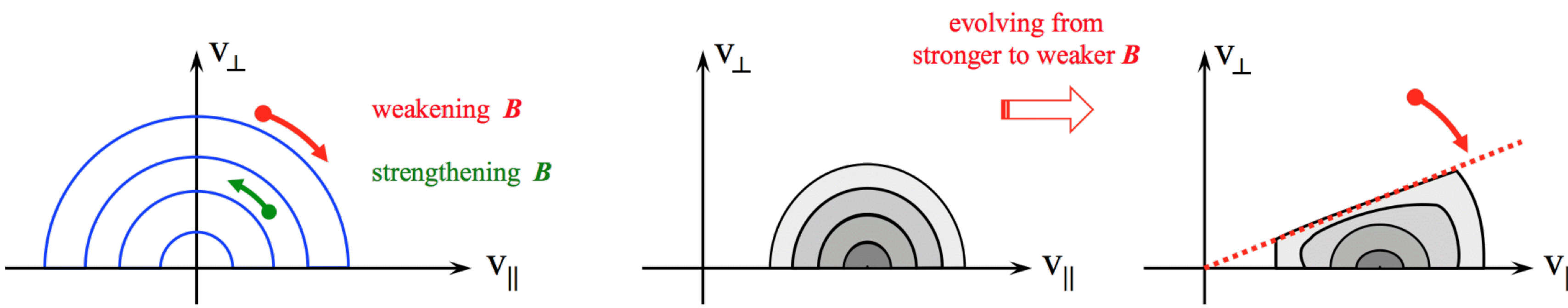


- However, the fact that we sometimes see $T_{\perp} > T_{\|}$ means there must exist sources of ongoing plasma heating that preferentially dump energy into the perpendicular (Larmor gyration) motions. There are **wave-particle resonances** that do this…

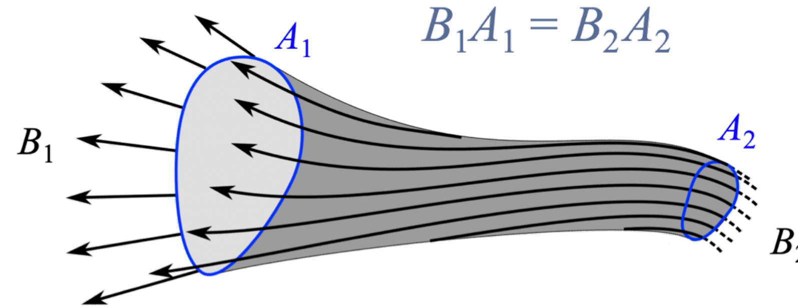
Alfvén wave oscillating E and B fields

ion's Larmor motion around radial B-field

(3b) *Non-Maxwellian distributions*

- What to use in the solar wind equations?
- First, let's be general about flux-tube expansion. The spherical limit is:

$$A(r) \propto r^2 \qquad \text{so} \qquad \frac{1}{A}\frac{dA}{dr} = \frac{2}{r}$$



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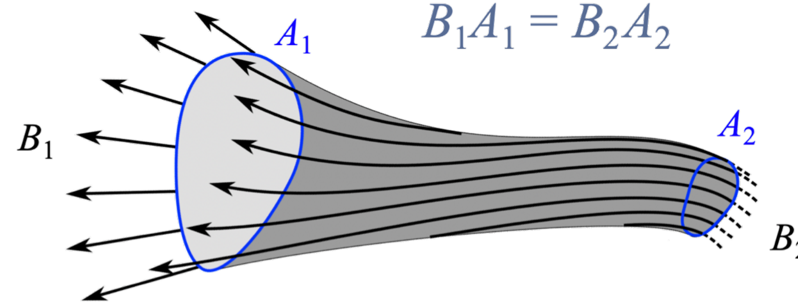

$$A(r) \propto r^2 \qquad \text{so} \qquad \frac{1}{A}\frac{dA}{dr} = \frac{2}{r}$$

- With that in mind, the Maxwellian (one-fluid) equation to replace is:

$$\left(u - \frac{c_i^2}{u}\right)\frac{du}{dr} = \frac{2c_i^2}{r} - \frac{dc_i^2}{dr} - \frac{GM_\odot}{r^2}$$

$$\Downarrow$$

$$\left(u - \frac{c_{\parallel,i}^2}{u}\right)\frac{du}{dr} = \frac{c_{\perp,i}^2}{A}\frac{dA}{dr} - \frac{dc_{\parallel,i}^2}{dr} - \frac{GM_\odot}{r^2}$$

(3c) *Superradial flux-tube expansion*

- A fraction of the Sun's surface is always covered by closed loops (at a given moment), but the volume of the solar wind (at large distances) fills a full 4π steradians.

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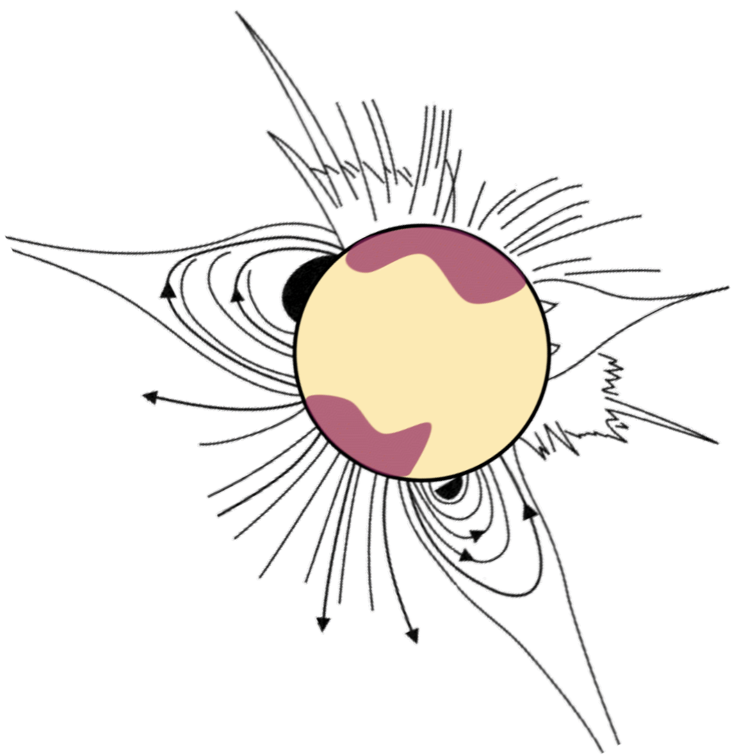
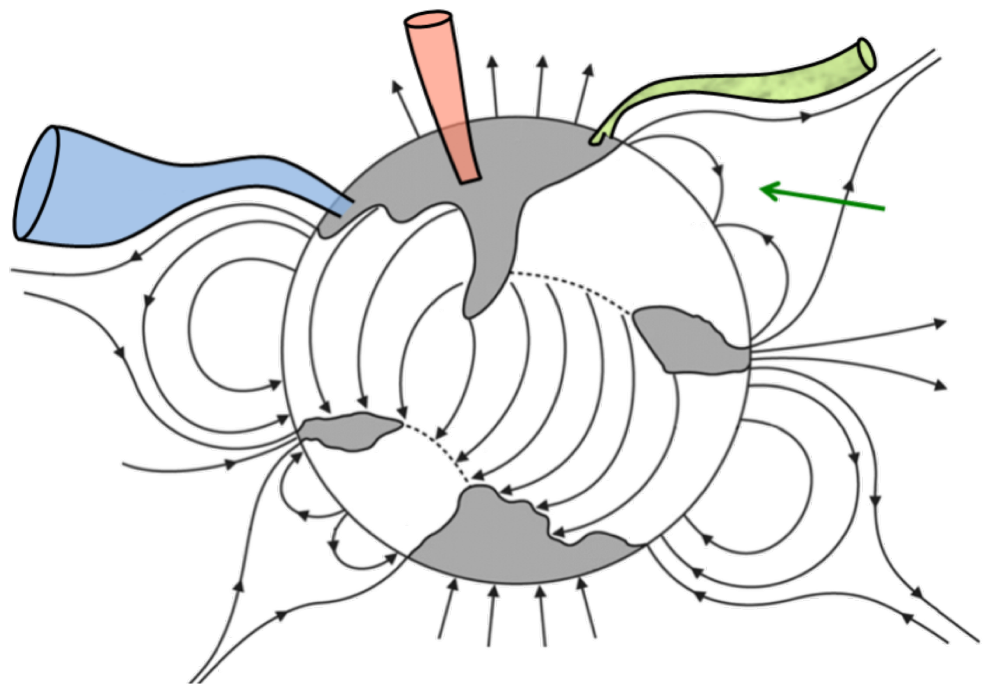
- A fraction of the Sun's surface is always covered by closed loops (at a given moment), but the volume of the solar wind (at large distances) fills a full 4π steradians.
- Thus, on average, the solar wind must flow through bundles of field lines that expand (i.e., "trumpet out") ***more*** than they would if they were spherically expanding "cones."

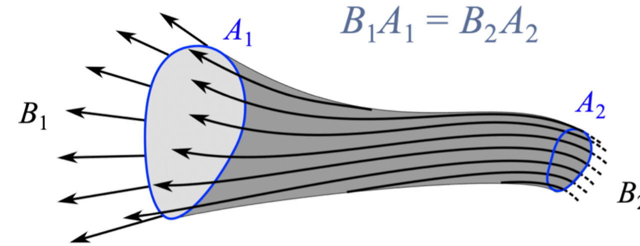




(3c) *Superradial flux-tube expansion*

- We've already described how the solar wind equations are modified for non-spherical expansion, but one often sees the spherical part separated out:

$$A(r) = r^2 f(r) \quad \text{so} \quad \frac{1}{A}\frac{dA}{dr} = \frac{2}{r} + \frac{1}{f}\frac{df}{dr}$$



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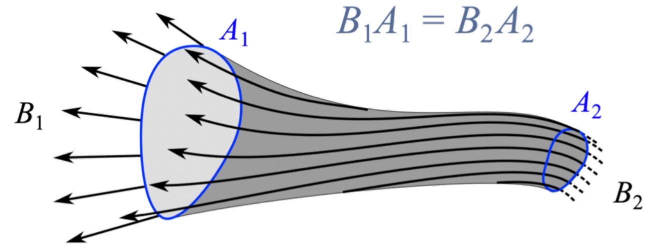

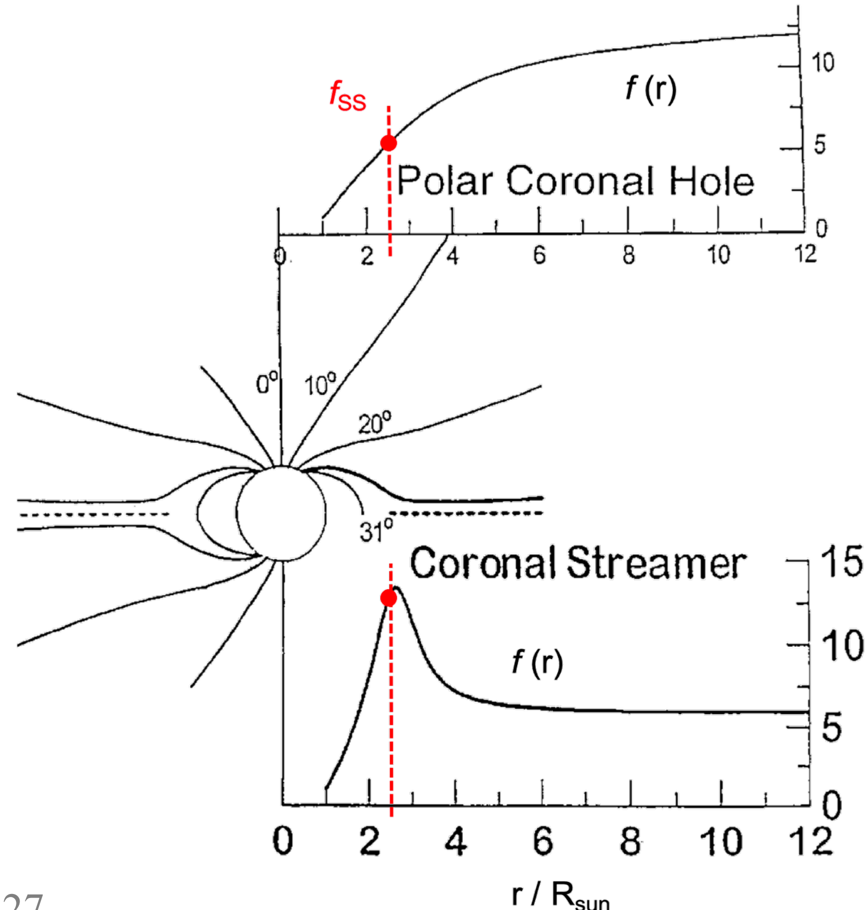

- Wang & Sheeley (1990) popularized taking the value of f **at the PFSS source surface** to characterize the overall level of expansion.

- Observations hint at anti-correlation between f_{SS} and wind speed at 1 AU.

(3d) “*Ponderomotive*” *wave pressure*

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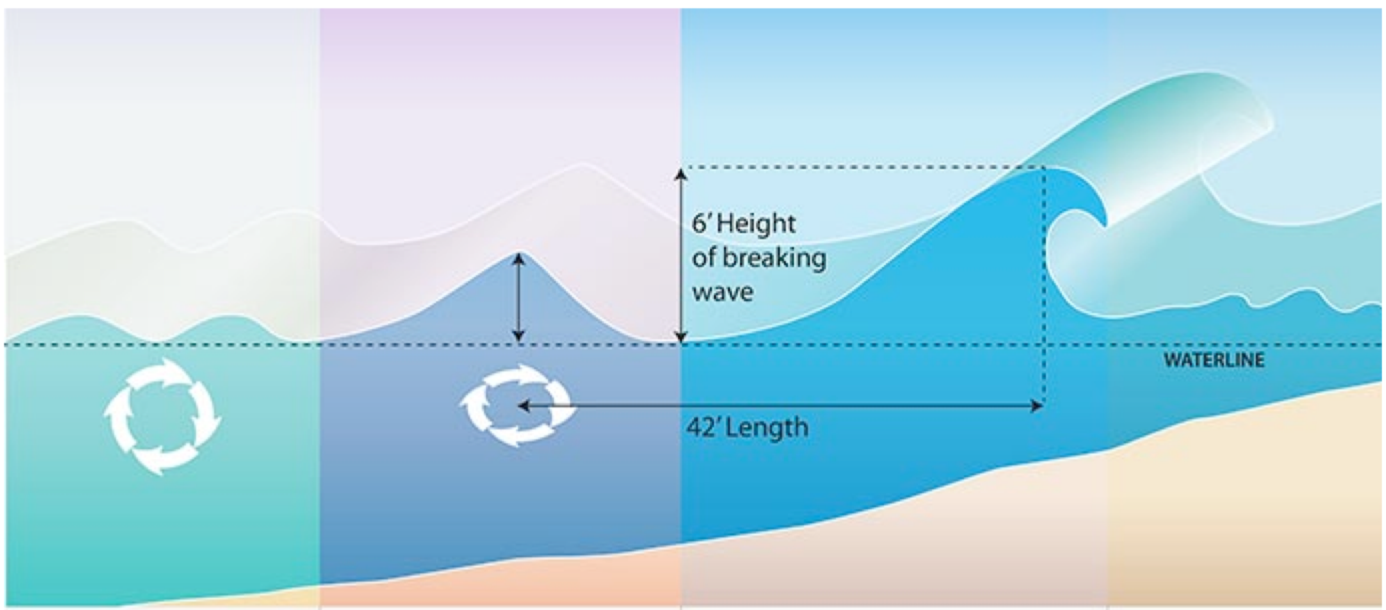
(3d) "*Ponderomotive*" *wave pressure*

- Just as electromagnetic waves carry momentum and exert pressure on matter, acoustic and MHD waves can **do work on the time-steady gas** via similar time-averaged stress.
- This depends on waves propagating through a spatially varying background medium, similar to how "breaking surf" at the beach can carry objects back to shore…

$$\left(u - \frac{c_i^2}{u}\right)\frac{du}{dr} = \frac{2c_i^2}{r} - \frac{dc_i^2}{dr} - \frac{GM_\odot}{r^2} - \frac{1}{\rho}\frac{d}{dr}\left[\frac{\rho\,\langle\delta v_\perp^2\rangle}{2}\right]$$



(3d) *"Ponderomotive" wave pressure*

- How does the Alfvén wave amplitude vary with radial distance?
- If we ignore dissipation, the conservation of wave energy flux gives

$$\langle \delta v_\perp^2 \rangle \propto \frac{u\, V_\mathrm{A}}{(u + V_\mathrm{A})^2}$$

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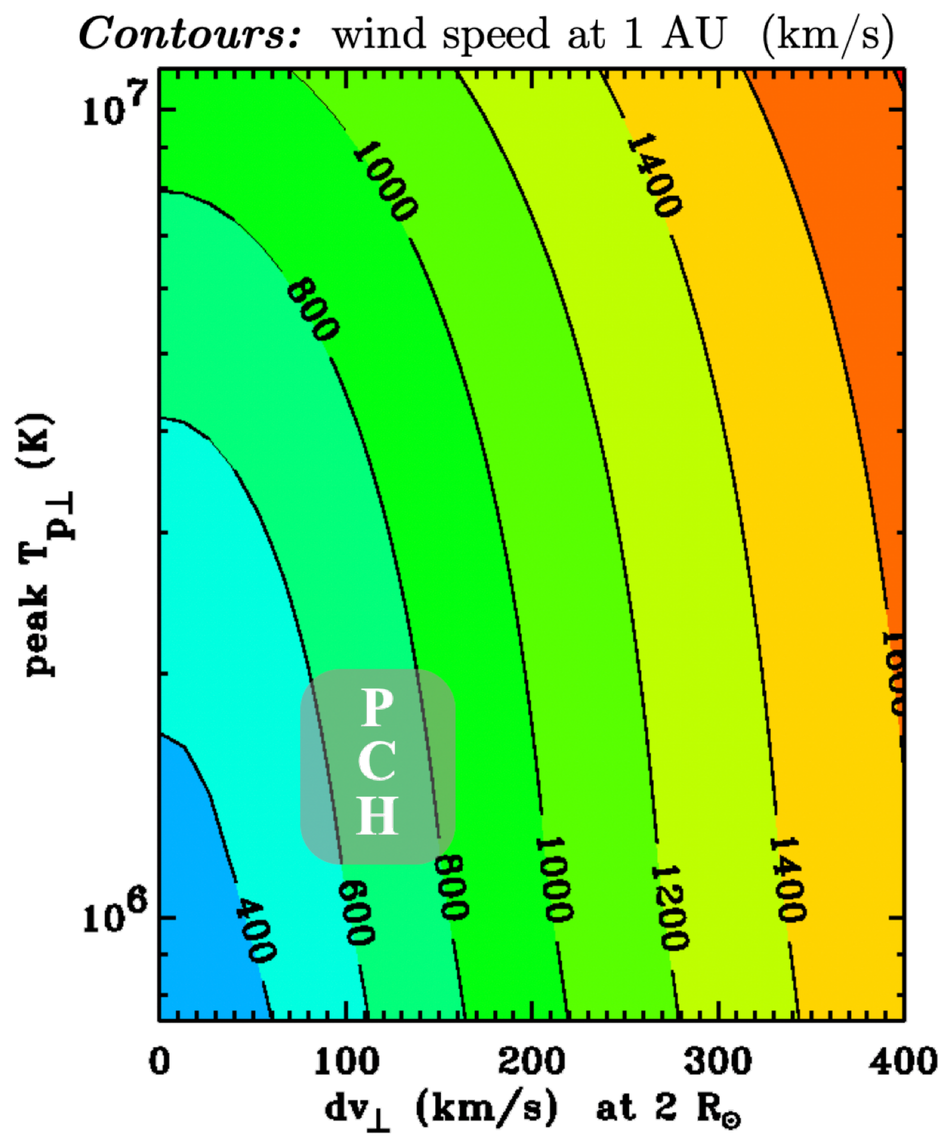
$$\langle \delta v_\perp^2 \rangle \propto \frac{u V_A}{(u + V_A)^2}$$

- A simple (parameterized) version of this expression was implemented for a grid of models with different proton temperatures and wave-amplitude normalizations.
- Observed proton temperatures in the corona are only 1–2 MK, so the gray **"Polar Coronal Hole"** region seems to pinpoint the parameter space needed to produce fast wind (600 to 800 km/s).



The python notebook

`handson_solarwind_v1.ipynb`

- Pinned at the top of the #hands-on-2-discussion channel on the Slack.
- Will be posted on course web page (under today's date in schedule).

For next week

- Work on your **extension** of what's in the notebook… e.g., answering questions posed therein, making modifications, adding new physics, re-implementing in a better programming language, etc… really whatever you would like to do that helps you understand coronal heating better.
- Due in 2 weeks (Thursday, March 10, 2022). Submit results to me via email.
- The dedicated Slack channel is for discussions about this work, but participation is not required.