



ASTR-6000 Seminar
COLLAGE: Coronal Heating,
Solar Wind, & Space Weather

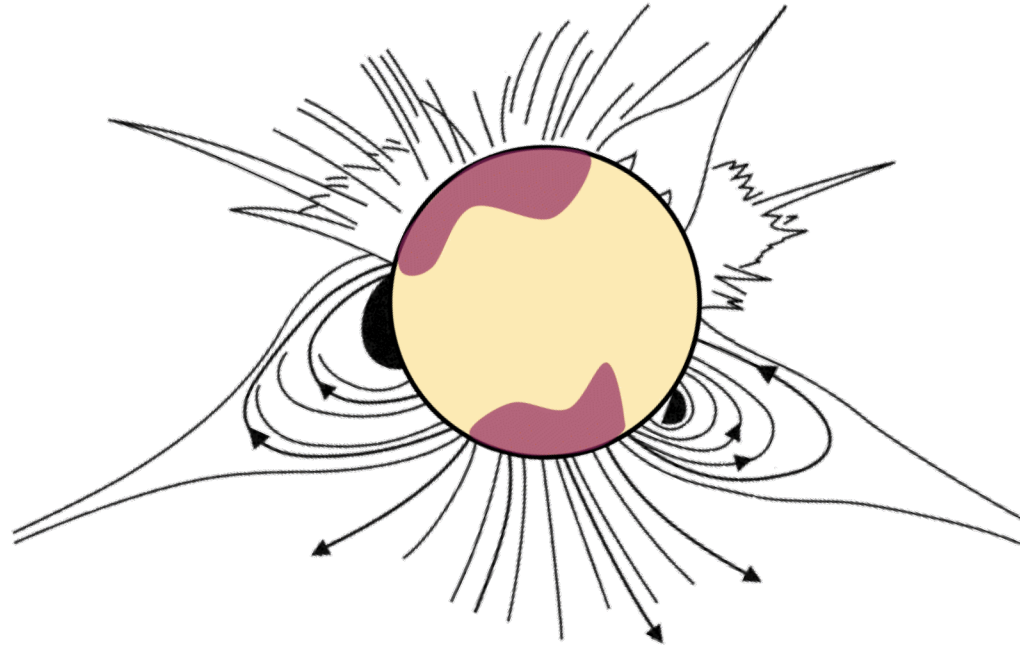
February 17, 2022

The open-field corona
& the discovery of the
solar wind

Dr. Steven R. Cranmer
Dr. Thomas E. Berger

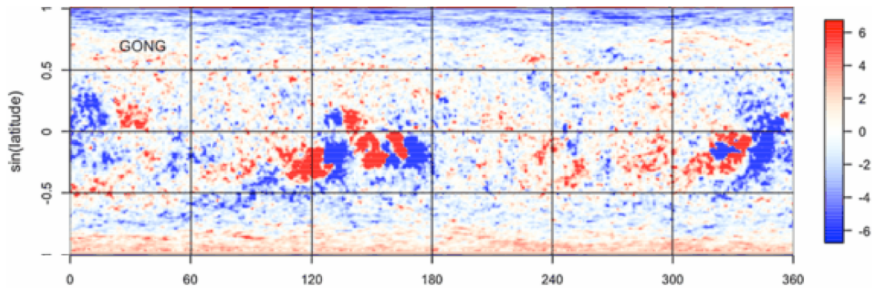
Outline

1. How does the Sun's magnetic field connect to the heliosphere?

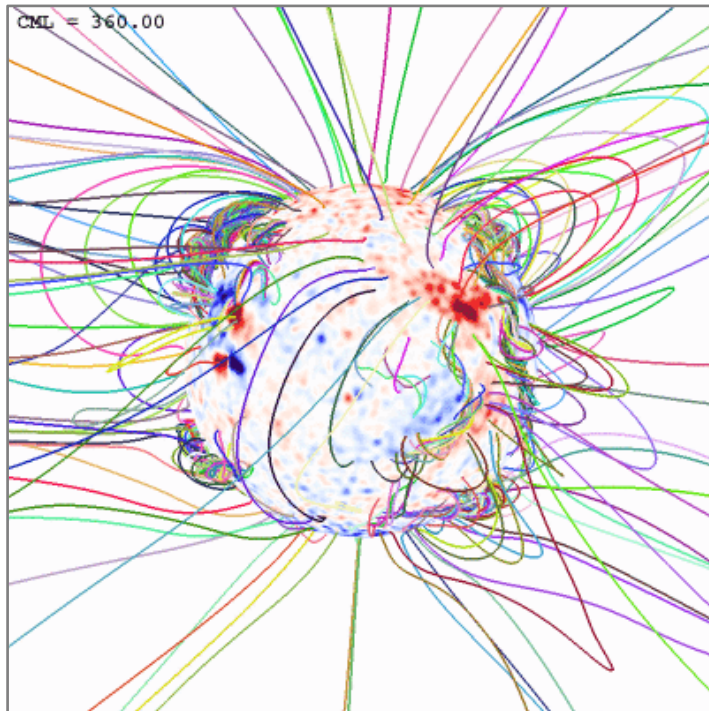


2. The solar wind: history & its physical necessity

(1) *The heliospheric magnetic field*



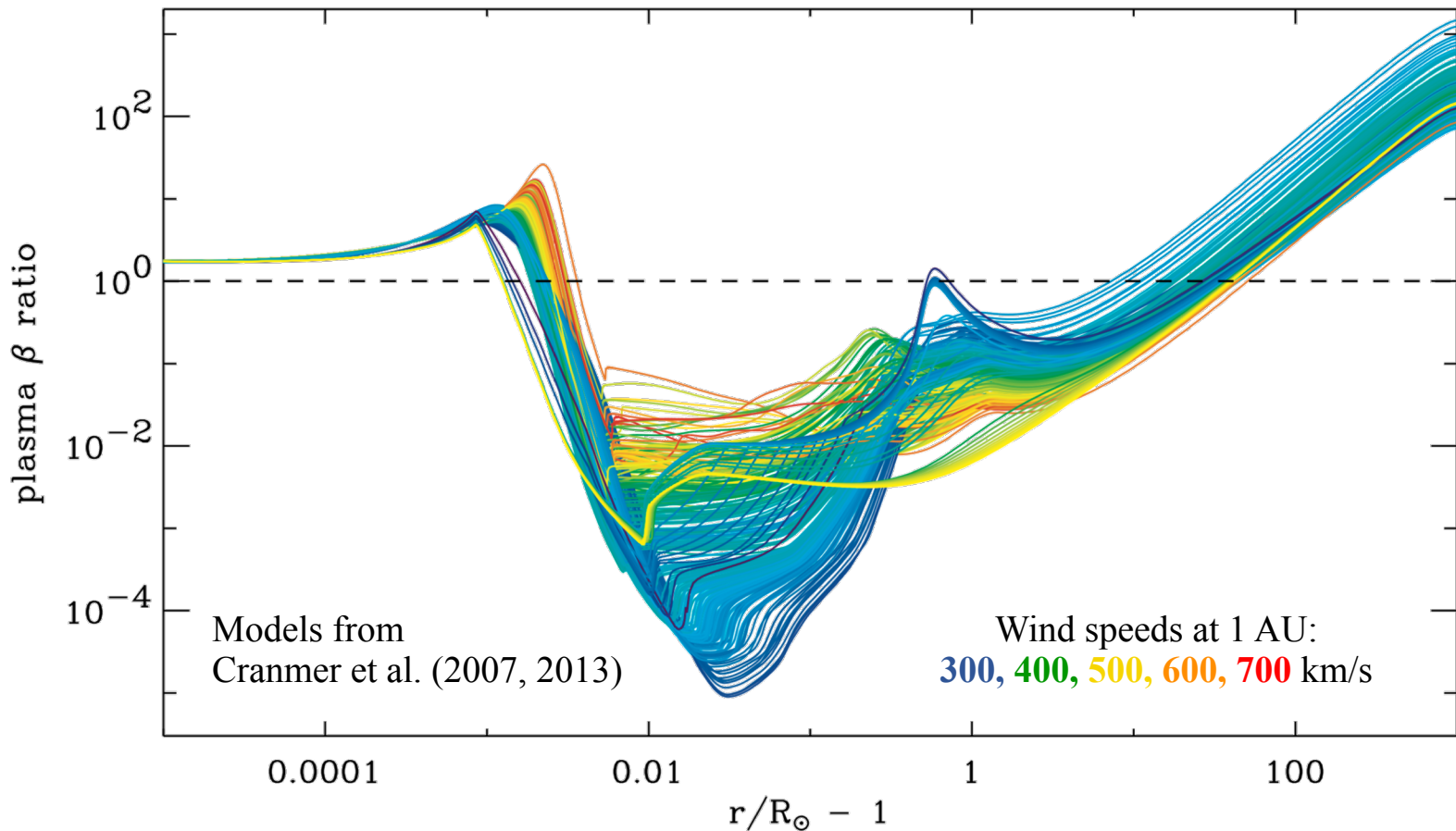
- Measurement of **photospheric B-field** is useful as a lower boundary condition.
- **Coronal B-field** measurements are difficult, but DKIST pushes the envelope...
- **In situ B** data helps constrain the total magnetic flux throughout the heliosphere.



(1) *The heliospheric magnetic field*

- There are multiple techniques for extrapolating the **B** field.
- All methods must deal with the fact that the corona is magnetically dominated, while the outer parts of the heliosphere are gas-pressure dominated...

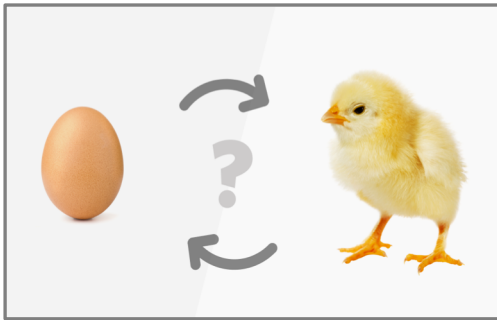
$$\beta = \frac{P_{\text{gas}}}{P_{\text{mag}}} \approx \left(\frac{c_s}{V_A} \right)^2$$



(1) *The heliospheric magnetic field*

- There are multiple techniques for extrapolating the **B** field...

(solar wind not taken into account)



- Potential-field solutions
- PFSS: Potential Field with Source Surface
- PFSS + current sheets
- Linear force-free fields
- Nonlinear force-free fields (e.g., magnetofrictional relaxation)
- Full solutions to 3D MHD equations



(fully self-consistent with the solar wind)



(1) *Simplified MHD in the corona*

- Momentum equation:
$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla P_{\text{gas}} + \mathbf{J} \times \mathbf{B} + \rho \mathbf{g}$$

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• Remember that $\nabla \times \nabla \psi = 0$ so we can express $\mathbf{B} = -\nabla \psi$

• “Potential fields” obey Laplace’s equation... $\nabla \cdot \nabla \psi = \nabla^2 \psi = 0$

(1) *The heliospheric magnetic field*

- Given a boundary condition on the sphere, the classical **potential field** is

$$\psi(r, \theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{+\ell} \left[a_{\ell m} \left(\frac{r}{R_{\odot}} \right)^{\ell} + b_{\ell m} \left(\frac{R_{\odot}}{r} \right)^{\ell+1} \right] Y_{\ell m}(\theta, \phi) \quad \mathbf{B} = -\nabla\psi$$

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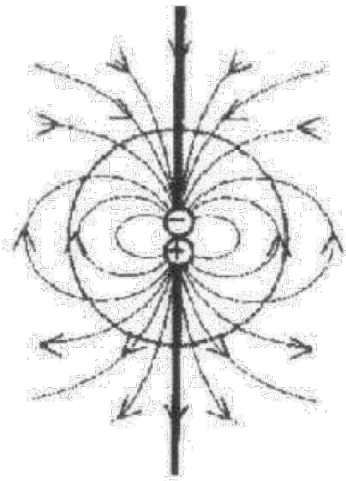
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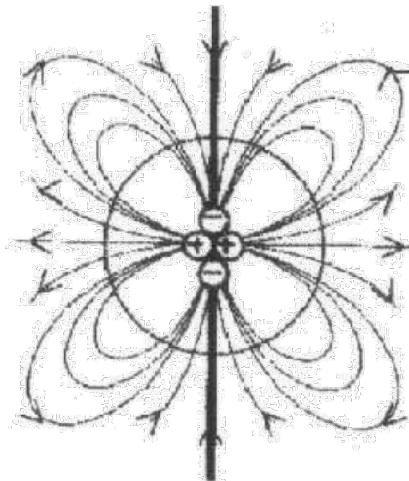
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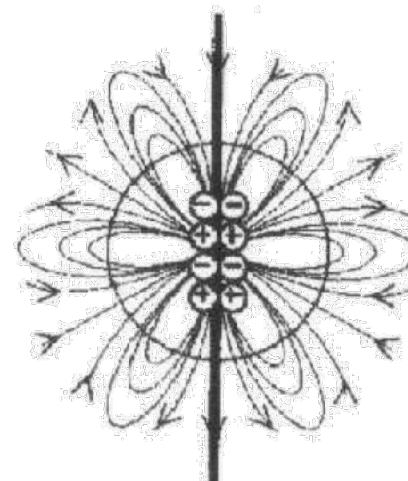
$\ell = 0$	monopole field	$\psi \propto r^{-1}$	\mathbf{B} drops off as $1/r^2$
$\ell = 1$	dipole field	$\psi \propto r^{-2}$	\mathbf{B} drops off as $1/r^3$
$\ell = 2$	quadrupole field	$\psi \propto r^{-3}$	\mathbf{B} drops off as $1/r^4$
$\ell = 3$	octupole field	$\psi \propto r^{-4}$	\mathbf{B} drops off as $1/r^5$



dipole



quadrupole



octupole

(1) *The heliospheric magnetic field*

- There's one problem with the standard potential field... it's closed.
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$$\psi = 0 \quad \text{for } r > R_{\text{ss}}$$

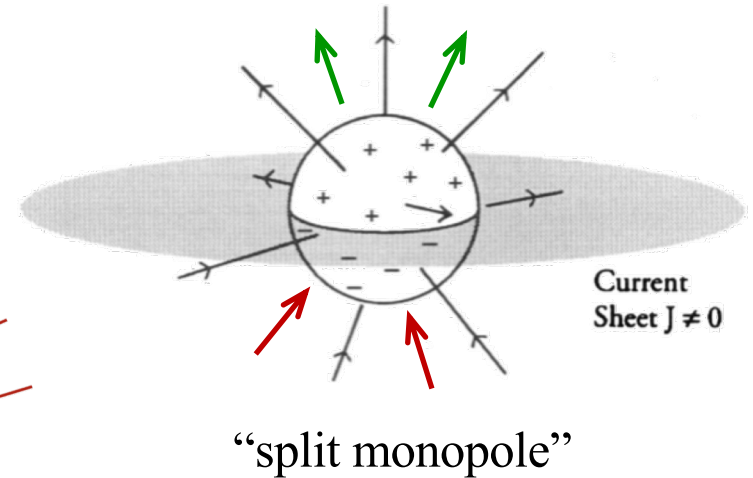
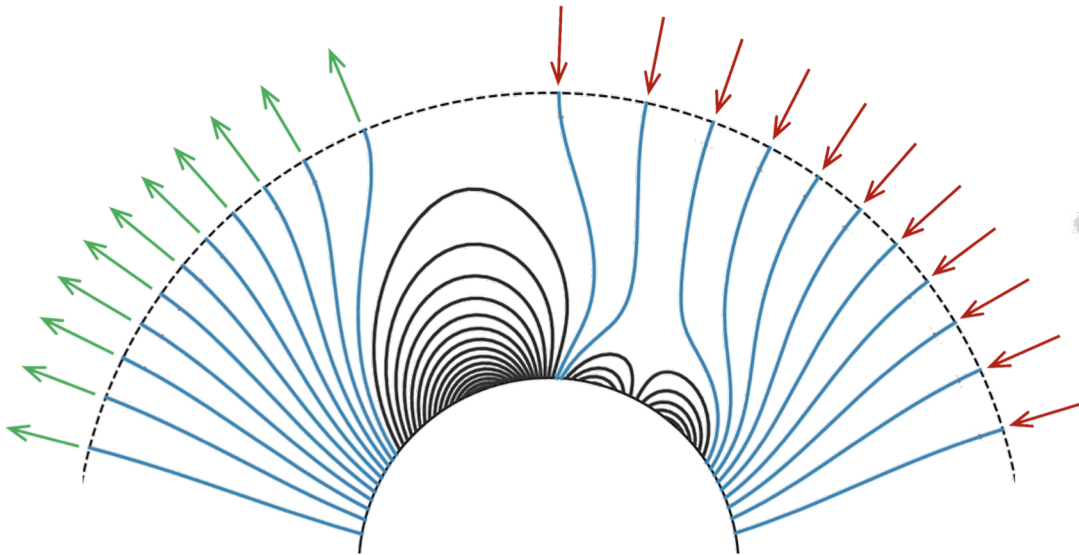
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- If $\psi = \text{constant}$ on the “source surface” ($r = R_{SS}$), then \mathbf{B} is purely radial at & above it.



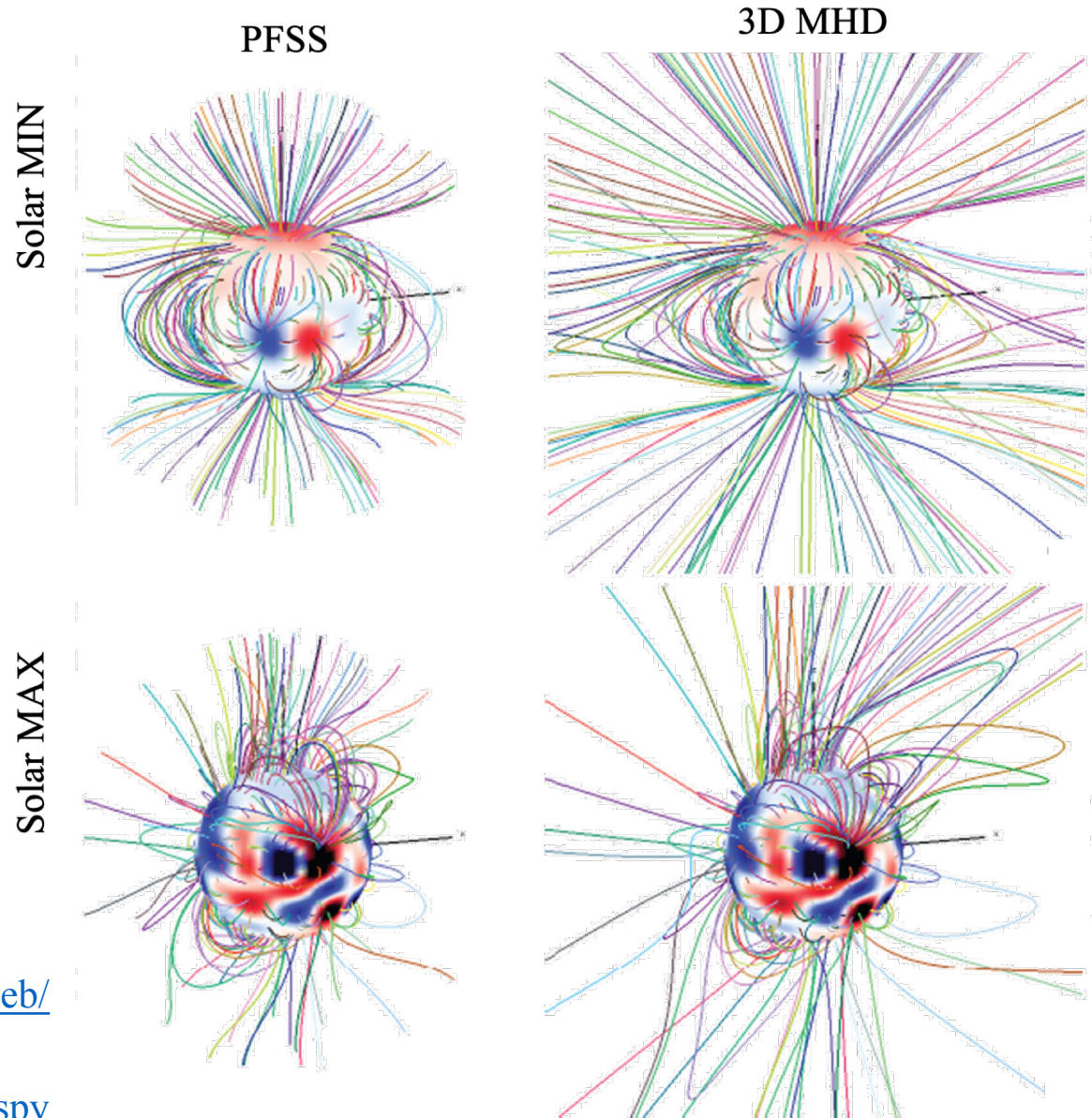
(1) *The heliospheric magnetic field*

- PFSS is fast!
- A choice of $R_{SS} = 2.5 R_{\odot}$ seems to capture many useful qualitative features of open/closed transitions in the outer corona, but no choice is perfect.
- Example comparisons from Riley et al. (2006) →
- See a recent non-potential model that attempts to do better than PFSS while avoiding the computational demands of MHD:
Rice & Yeates (2021, *ApJ*, 923, 57).

<https://www.predsci.com/mhdweb/>

<https://ccmc.gsfc.nasa.gov/>

<https://github.com/dstansby/pfsspy>

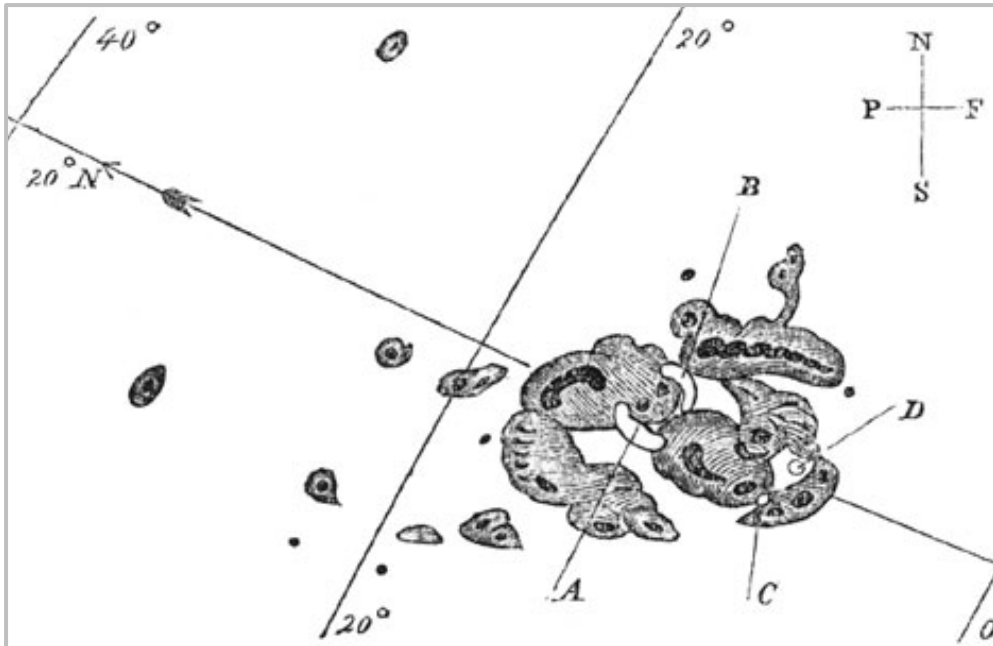


(2) *The solar wind*

- **Late 1800s → early 1900s:** evidence kept accumulating for *something* out there that connects the Sun and the Earth together.
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- **Late 1800s → early 1900s:** evidence kept accumulating for *something* out there that connects the Sun and the Earth together.
- I'll summarize 3 pieces of circumstantial evidence that Parker (1958) had in-hand...
- First, after the “Carrington event” (Sept. 1, 1859), there was increased awareness that there's some kind of cause-and-effect between events on the Sun & events on Earth.



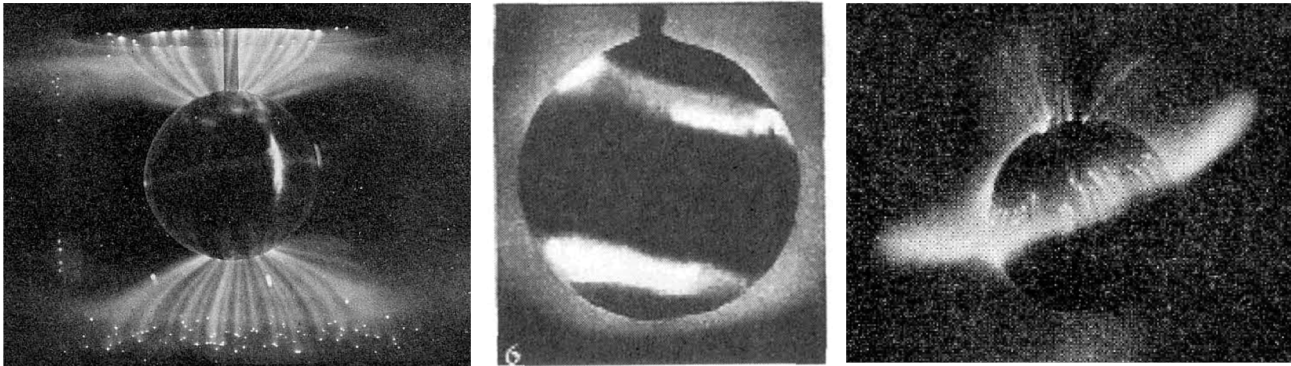
A. STORM OF ELECTRICITY

TELEGRAPH WIRES USELESS FOR
SEVERAL HOURS.

ONE OF THE MOST SEVERE DISTURBANCES
FOR MANY YEARS, EXTENDING EVEN TO
EUROPE—TELEPHONE WIRES ALSO OB-
STRUCTED—BUSINESS DELAYED A GOOD
PART OF THE DAY.

(2) *The solar wind*

- Ennis (1878): solar corona & terrestrial aurora were both the same kind of *“streaming forth of electricity.”*
- Birkeland (1908): aurorae & geomagnetic storms *“...should be regarded as manifestations of an unknown cosmic agent of solar origin.”*



(and he reproduced many auroral phenomena in the lab!)

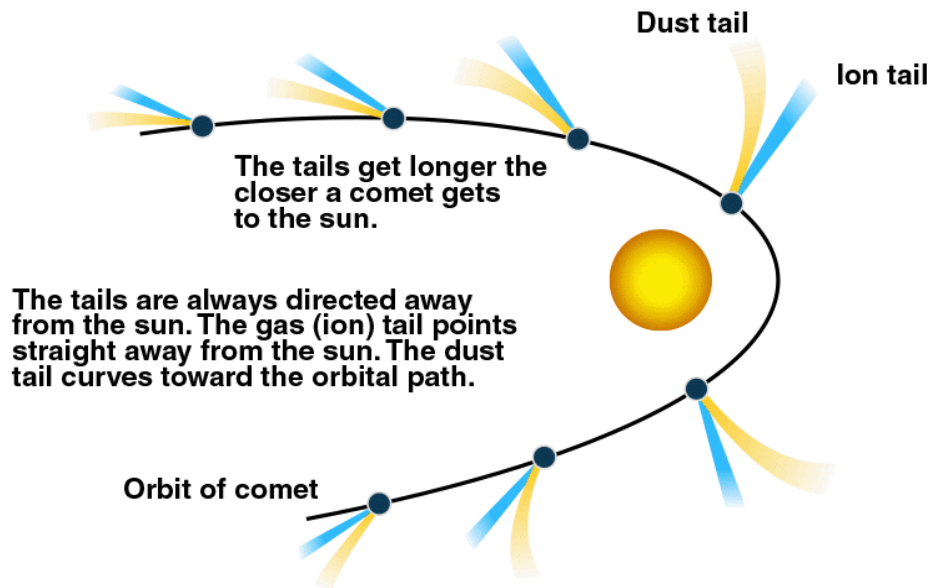
- Serviss (1909): compared polar plumes to field lines traced by iron filings; hypothesized *“...there exists a direct solar influence not only upon the magnetism, but upon the weather of the earth.”*
- Chapman (1918) proposed the Sun emits sporadic clouds of charged particles into the vacuum of space.

(for citations, see [Cranmer 2019](#))

(2) *The solar wind*

- Second piece of evidence: many comets have **dust tails** (whose ejecta fall back along ballistic orbits) and **ion tails** (always oriented away from the Sun).
- Biermann (1951) also analyzed the kinematics of ion-tail inhomogeneities, which can be tracked to flow away from the Sun at speeds of a few $\times 100$ km/s.

Kometenschweife und solare Korpuskularstrahlung (Comet tails and solar corpuscular radiation)



(2) *The solar wind*

- Third piece of evidence: the existence of the hot (10^6 K) corona was well known, but if the corona was assumed to exist in a state of **hydrostatic equilibrium**, it gives a nonsensical answer for the gas pressure.
- For Chapman's (1957) heat conduction model,

$$T(r) = T_0 \left(\frac{r_0}{r} \right)^{2/7} \quad \text{and as } r \rightarrow \infty, \quad P \rightarrow P_0 \exp \left[- \left(\frac{V_{\text{esc},0}}{c_{s,0}} \right)^2 \right] \approx 10^{-5} \text{ dynes/cm}^2$$

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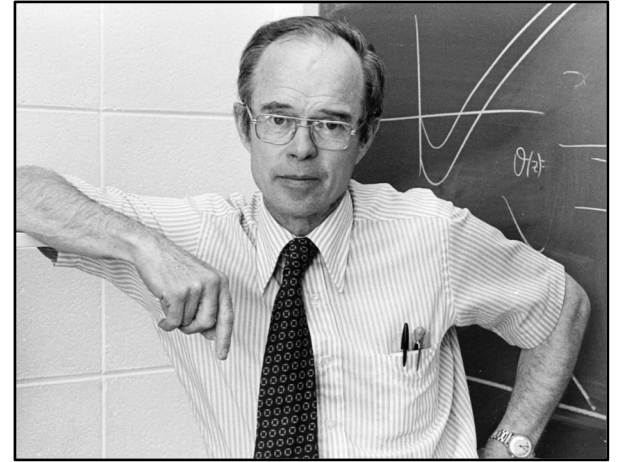
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- However, by the 1950s, astronomers already knew typical gas pressures in the local **interstellar medium** were only 10^{-14} to 10^{-12} dynes/cm².
- If such a huge pressure difference were to exist, the pressure-gradient force would cause the corona to expand (explosively?) out to huge distances (parsecs?) before coming into equilibrium with the interstellar medium.
- If a new hydrostatic equilibrium was established, the near-Sun corona wouldn't look anything like it does now...
 - Something about this doesn't make sense.

(2) *The solar wind*

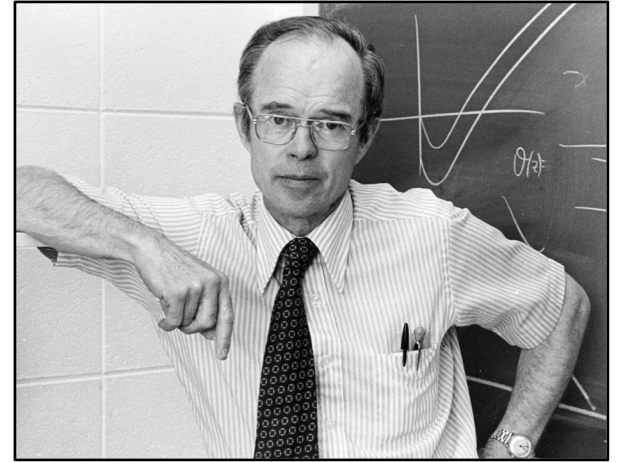
- Eugene Parker (1958a, *ApJ*, 128, 664) put the pieces together to conclude the gas in the outer corona must be in a continual state of acceleration & expansion.
- **2018**: first time NASA names a spacecraft after a living scientist: *Parker Solar Probe!*
- https://www.youtube.com/watch?v=WH_TC9VzMUA



Parker's idea was controversial at first... but **1958** was the dawn of the space age...

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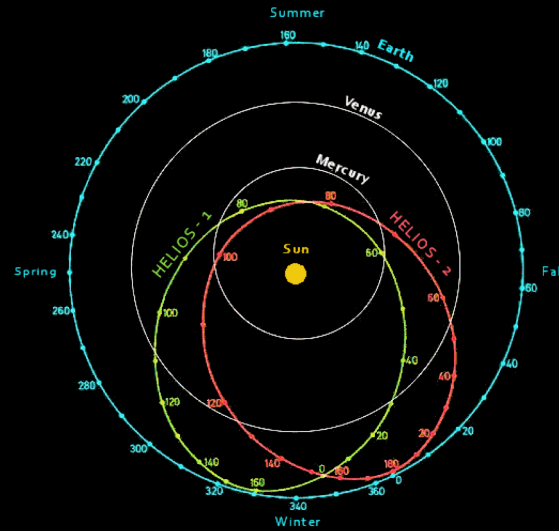
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- **1959-1961**: Intermittent measurements on early probes that left Earth's magnetosphere: Russian *Lunik*, *Venera*; American *Explorer 10*
- **1962**: Marcia Neugebauer & colleagues got continuous data from *Mariner 2* on its way to Venus.
- The solar wind exists, and it behaves very much like Parker predicted.

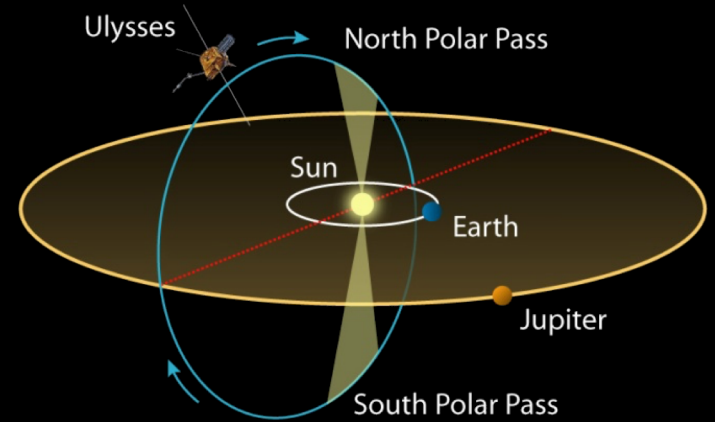


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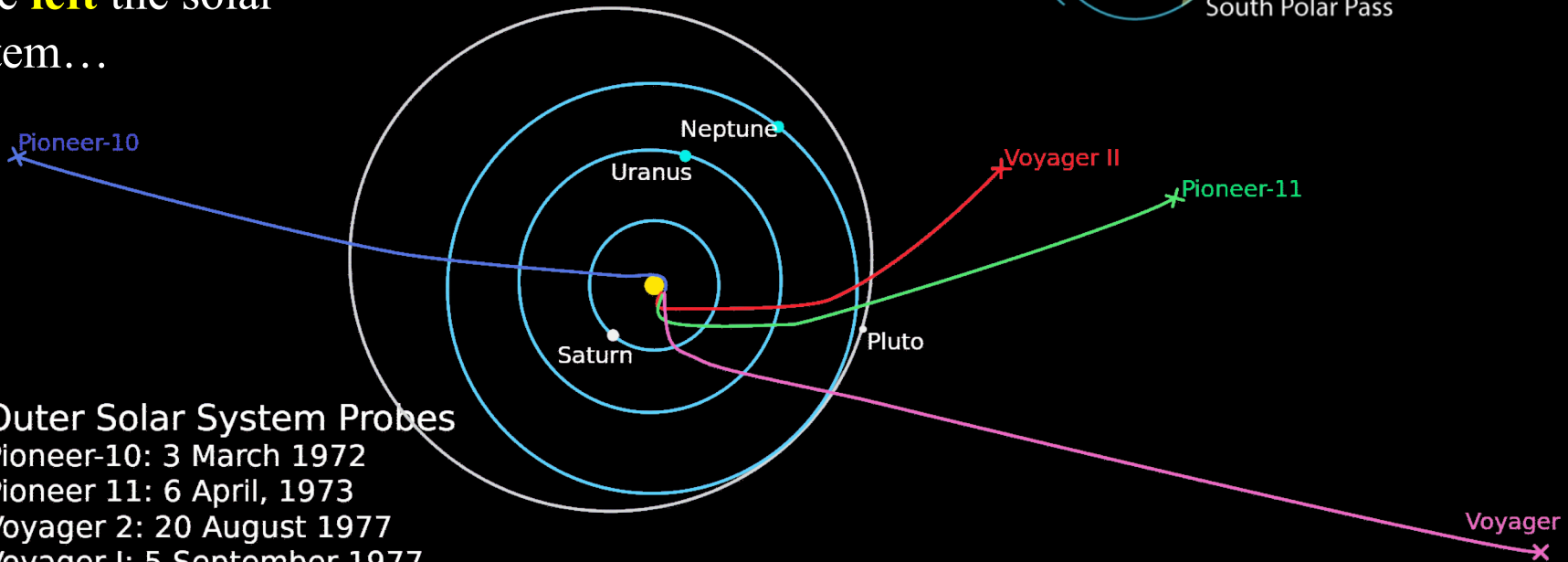
Helios & PSP went **in** past Mercury's orbit



Ulysses went **up & out** of the ecliptic plane



Voyagers & Pioneers have **left** the solar system...



Outer Solar System Probes

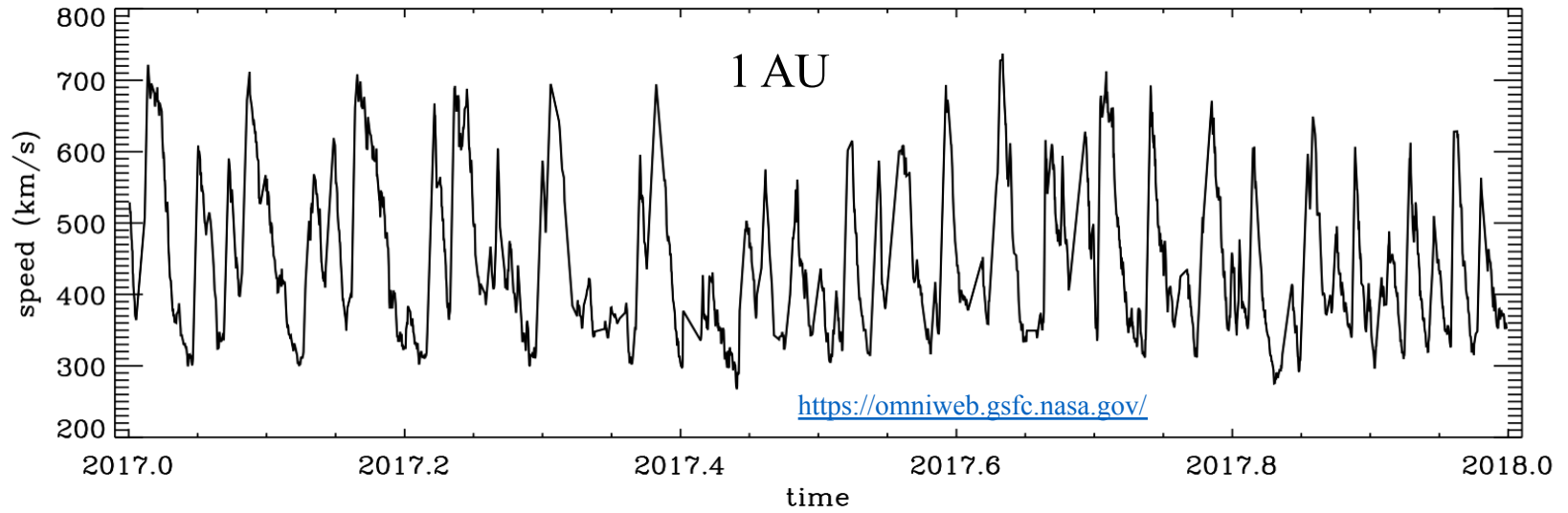
Pioneer-10: 3 March 1972

Pioneer 11: 6 April, 1973

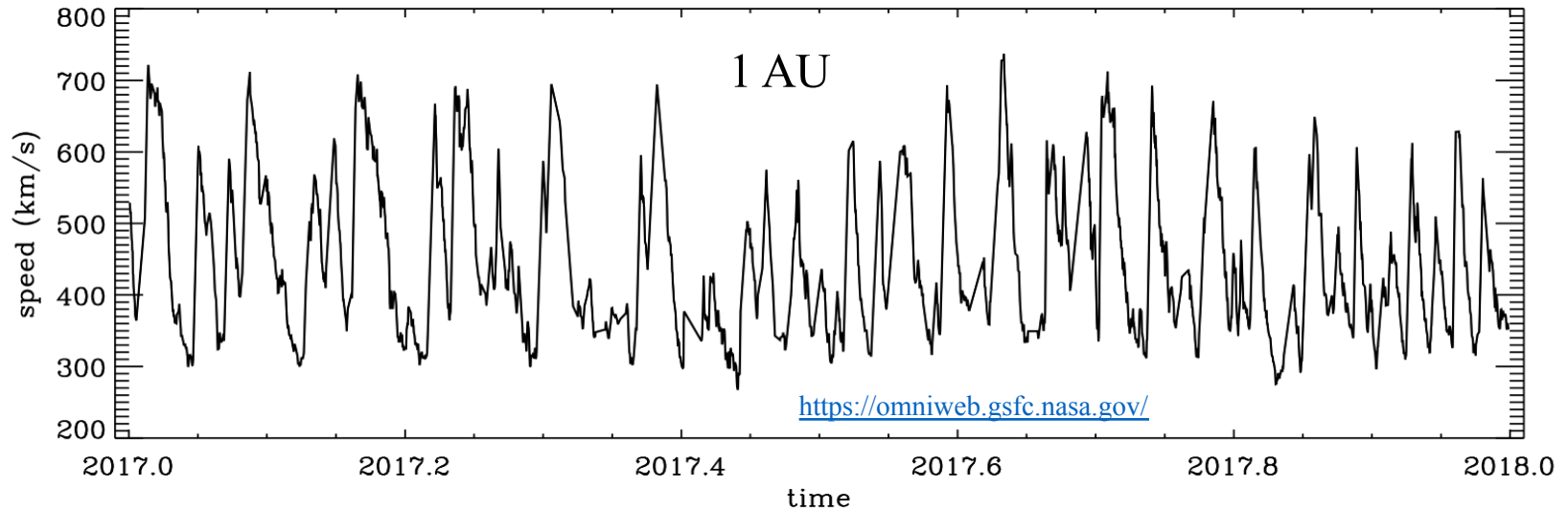
Voyager 2: 20 August 1977

Voyager I: 5 September 1977

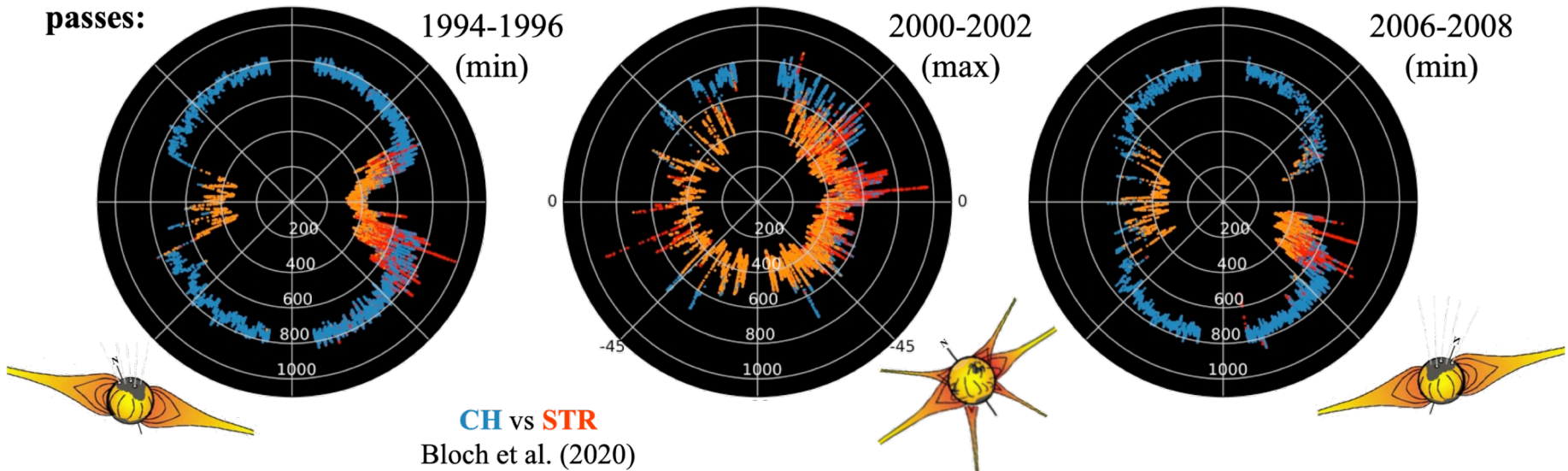
(2) *The solar wind*



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Ulysses polar passes:



(2) *The solar wind*

Parker's initial hydrodynamic solution:

- Time-steady (i.e., all time derivatives = 0), but still allowing $\mathbf{u} \neq 0$
- Spherically symmetric expansion (consistent with $B_r \propto 1/r^2$, but no MHD needed)
- Isothermal ($T = \text{constant}$, assumes conduction keeps it from varying much)

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Mass conservation:
$$\frac{1}{r^2} \frac{d}{dr} (r^2 \rho u) = 0$$

Momentum conservation:
$$u \frac{du}{dr} = -\frac{1}{\rho} \frac{dP}{dr} - \frac{GM_{\odot}}{r^2}$$

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$$\text{Momentum conservation:} \quad u \frac{du}{dr} = -\frac{1}{\rho} \frac{dP}{dr} - \frac{GM_{\odot}}{r^2}$$

For a time-steady system, mass conservation is simple:

$$r^2 \rho u = \text{constant.} \quad \text{By convention, we define } \boxed{\dot{M} = 4\pi \rho u r^2}$$

and \dot{M} is the total **mass loss rate** of the wind (in kg/s), integrated around all 4π steradians of solid angle.

- **Note:** Parker (1958) doesn't predict \dot{M} . For that, we need the “base pressure” from a coronal heating model like RTV or Martens (2010)!

(2) *The solar wind*

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$$u \frac{du}{dr} = -\frac{1}{\rho} \frac{dP}{dr} - \frac{GM_{\odot}}{r^2}$$

$$\frac{dP}{dr} = \left(\frac{k_B T}{\mu m_H} \right) \frac{d\rho}{dr} = c_i^2 \frac{d\rho}{dr} = c_i^2 \left(-\frac{\rho}{u} \frac{du}{dr} - \frac{2\rho}{r} \right)$$

(from mass conservation)

c_i = “isothermal” sound speed

- The RHS is a sum of outward (pressure gradient) + inward (gravity) forces.
- The LHS can be counter-intuitive... does $RHS > 0$ produce radial acceleration or deceleration? Compressible “transonic” flows can be weird.

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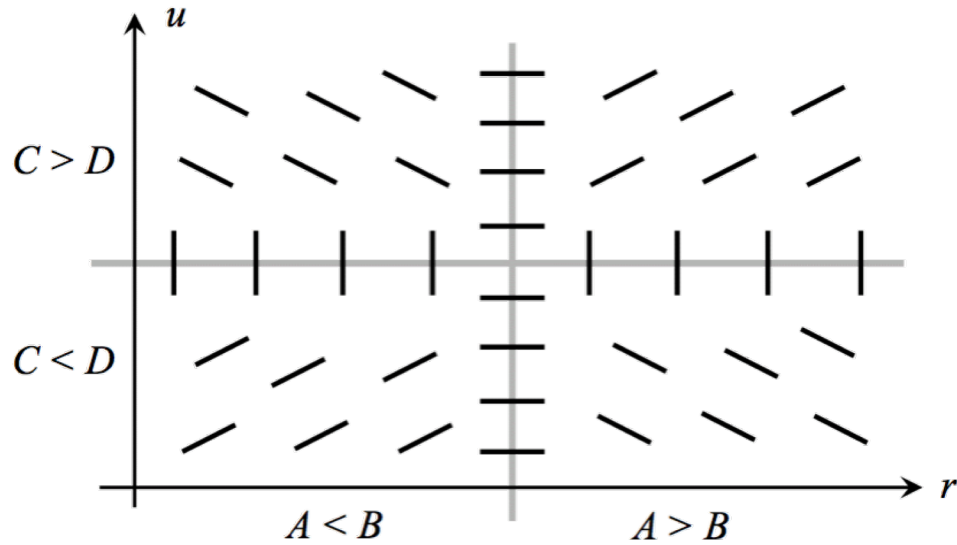
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$$\left(u - \frac{c_i^2}{u} \right) \frac{du}{dr} = \left(\frac{2c_i^2}{r} - \frac{GM_{\odot}}{r^2} \right) \quad \Rightarrow \quad \frac{du}{dr} = \frac{A(r) - B(r)}{C(u) - D(u)}$$

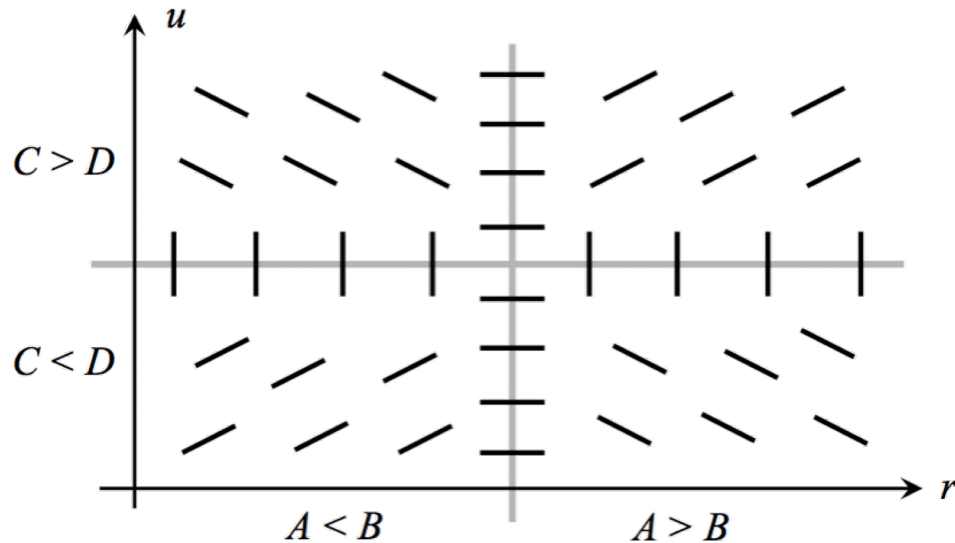
(2) *The solar wind*



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Note there are **singular points** when:

- $A = B$, but $C \neq D$ (flat)
- $A \neq B$, but $C = D$ (unphysical?)
- Both $A = B$ and $C = D$
(Parker's critical point)

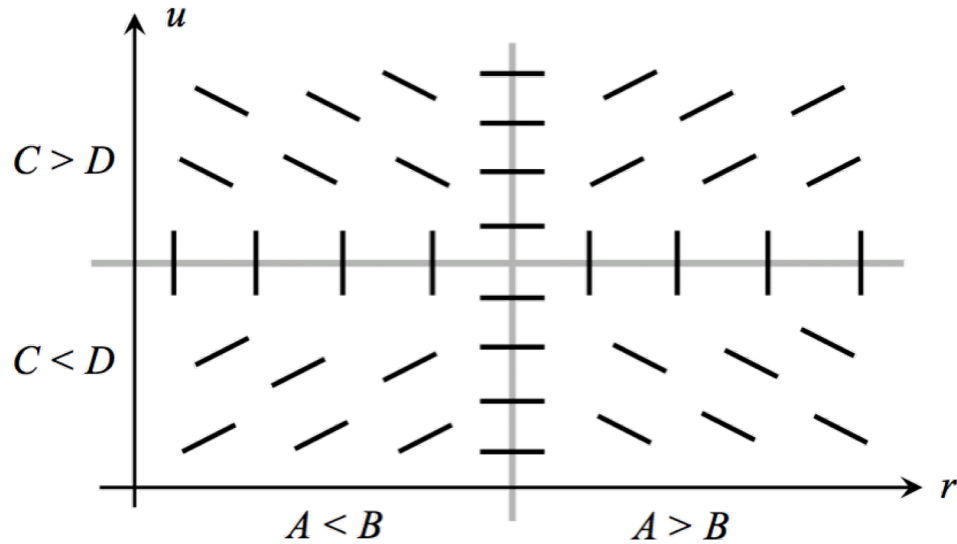
(0/0, so L'Hôpital's rule gives du/dr)

$$r_c = \frac{GM_\odot}{2c_i^2}$$

$$u_c = c_i$$

For a larger c_i , the critical point is closer to the Sun (∇P "beats" gravity sooner).

(2) The solar wind



$$\frac{du}{dr} = \frac{A(r) - B(r)}{C(u) - D(u)}$$

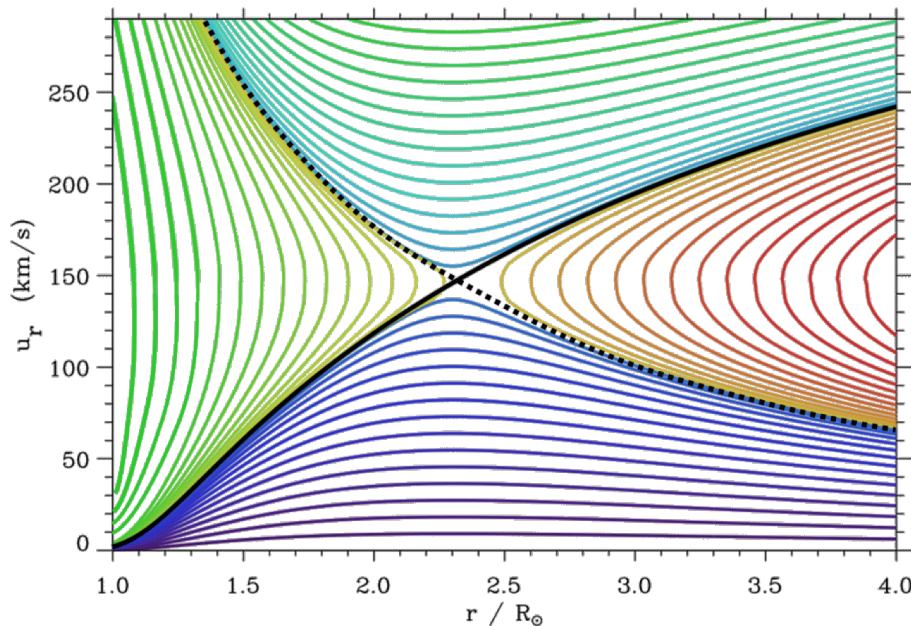
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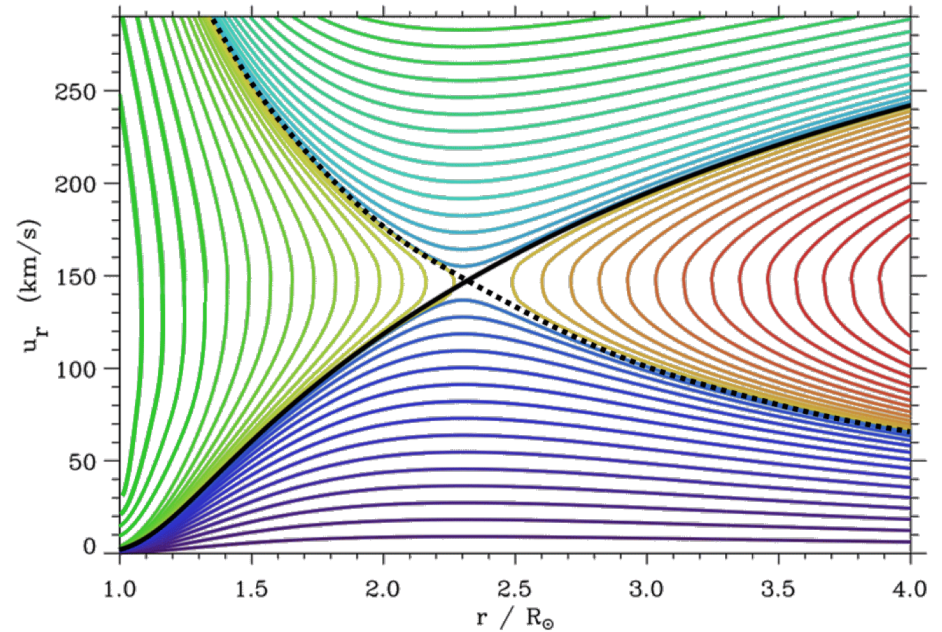
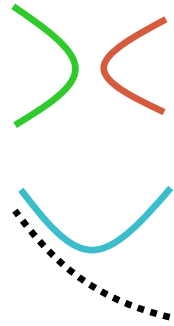
For a larger c_i , the critical point is closer to the Sun (∇P "beats" gravity sooner).



(2) *The solar wind*

Which solutions correspond to reality?

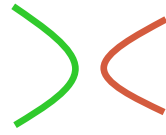
- Some are double-valued
- Some have fast outflows at the Sun's surface (where we don't really see outflows)
- What's left?



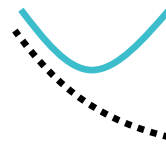
(2) *The solar wind*

Which solutions correspond to reality?

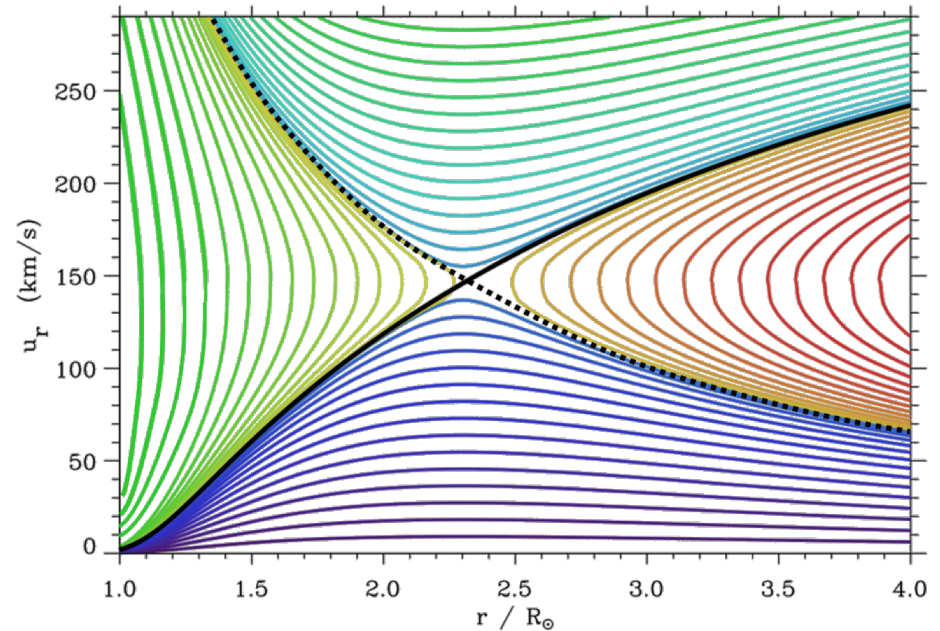
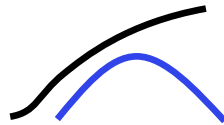
- Some are double-valued



- Some have fast outflows at the Sun's surface (where we don't really see outflows)



- What's left?



- **The subsonic solar breezes**, which have $u \rightarrow 0$ at infinity, so they suffer the same problems as the hydrostatic solutions.
- **The “transonic” solar wind**, which Parker suspected was the true steady-state “stable attractor” (proved by Velli 1994, *ApJL*, 432, L55)

$$(u^2 - c_i^2) - c_i^2 \ln \left(\frac{u^2}{c_i^2} \right) = 4c_i^2 \ln \left(\frac{r}{r_c} \right) + 2GM_\odot \left(\frac{1}{r} - \frac{1}{r_c} \right)$$

(2) *The solar wind*

- Parker (1958): higher temperature, faster wind...

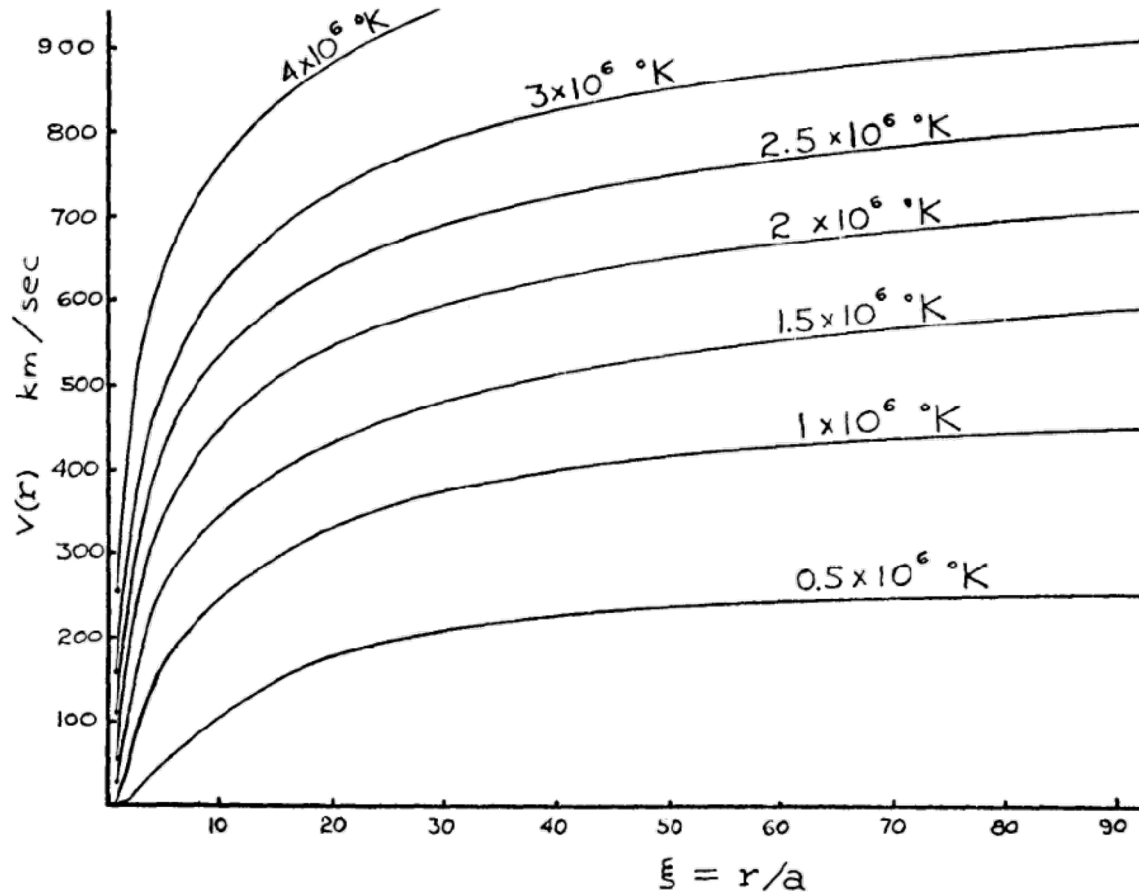


FIG. 1 —Spherically symmetric hydrodynamic expansion velocity $v(r)$ of an isothermal solar corona with temperature T_0 plotted as a function of r/a , where a is the radius of the corona and has been taken to be 10^{11} cm

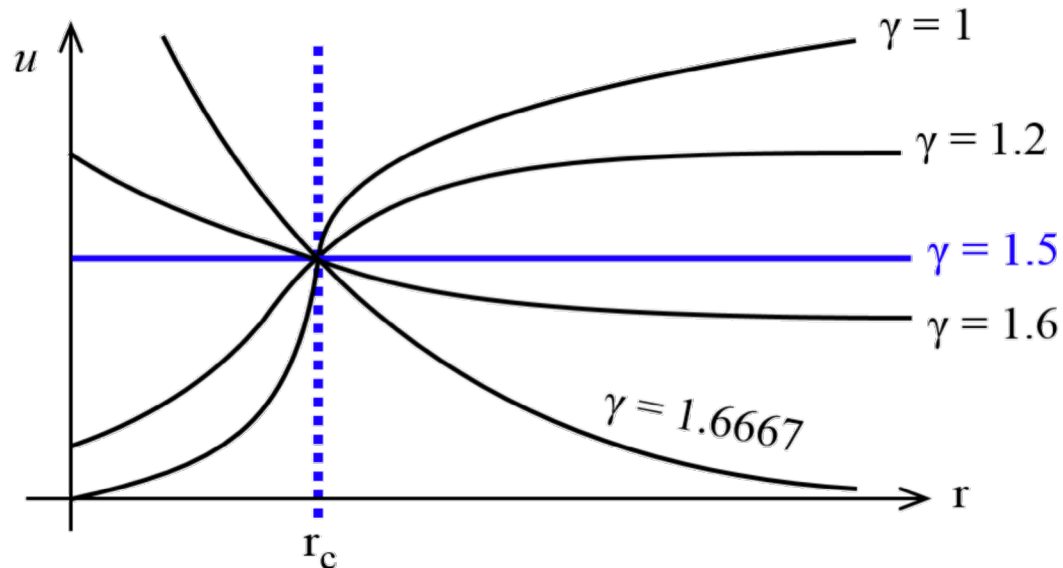
(2) *The solar wind*

- Owocki (2004) outlined another way to solve for $u(r)$ via Bernoulli's equation...

$$\mathcal{E} = \frac{u^2}{2} - \frac{GM_{\odot}}{r} + \left(\frac{\gamma}{\gamma - 1} \right) c_i^2 = \text{constant} .$$

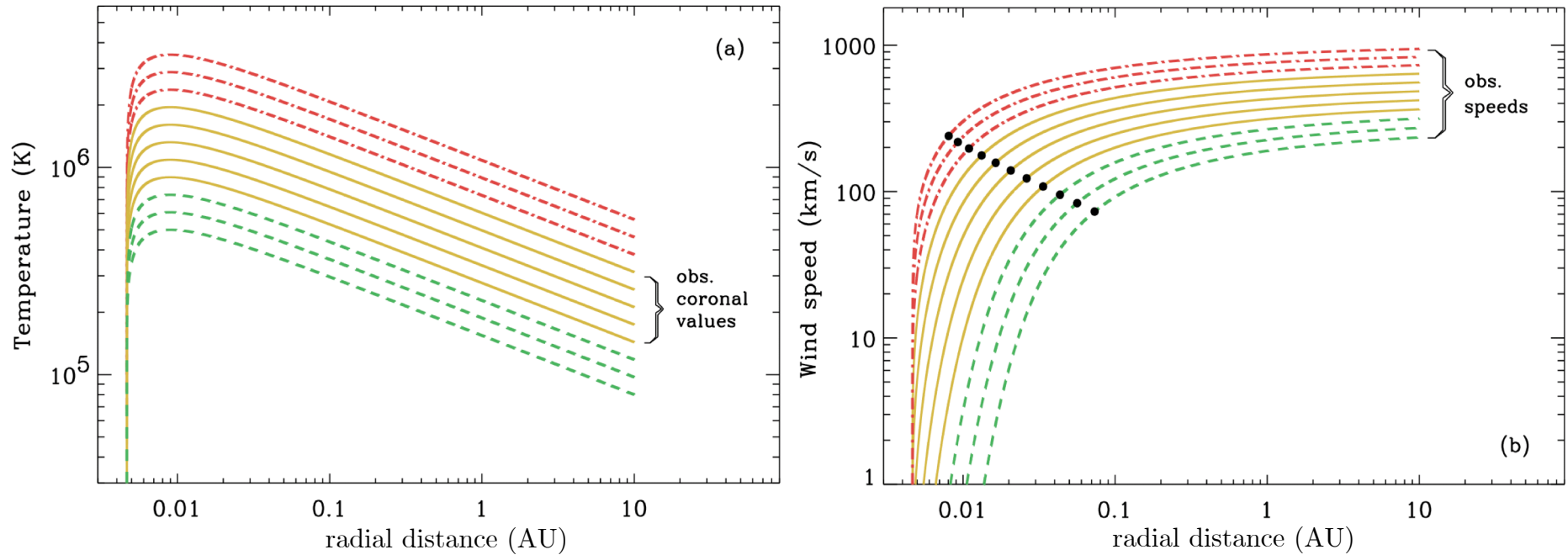
$$\mathcal{E}_c = \frac{u_c^2}{2} \left(\frac{5 - 3\gamma}{\gamma - 1} \right) \quad \mathcal{E}_{\infty} = \frac{u_{\infty}^2}{2} \quad \text{i.e.,} \quad \boxed{u_{\infty} = u_c \sqrt{\frac{5 - 3\gamma}{\gamma - 1}}}$$

- Accelerating solutions with a constant “terminal speed” require $1 < \gamma < 1.5$.
- As $r \rightarrow \infty$, adiabatic expansion has $T \propto r^{2-2\gamma}$, *in situ* data constrains $1.15 < \gamma < 1.35$.



(2) *The solar wind*

- If we have an observationally-constrained numerical tabulation for $T(r)$, we can just integrate the momentum equation numerically...



- **Problem:** the full range of observed temperatures (yellow) reproduces only *some* of the observed speeds at 1 AU.
- Other physics is needed to explain slowest ($u < 300$ km/s) & fastest ($u > 600$ km/s) solar wind streams.

For next week

Read (parts of) two complementary papers:

- Paper 4a: Wang & Sheeley (1990, *ApJ*, 355, 726, 8 pages),
“Solar wind speed and coronal flux-tube expansion”
[okay to skip: section **IV**] [[ADS](#)]
- Paper 4b: Cranmer (2005, arXiv:astro-ph/0506508, 6 pages),
“Why is the fast solar wind fast & the slow solar wind slow?”
[okay to skip: sections **5, 6, and 7**] [[ADS](#)]
- Participate in the [#paper-4-discussion](#) channel on Slack for next week
(February 24, 2022)