ASTR-6000 Seminar
COLLAGE: Coronal Heating, Solar Wind, & Space Weather

February 3, 2022

Coronal heating: waves & turbulence (a whirlwind tour)

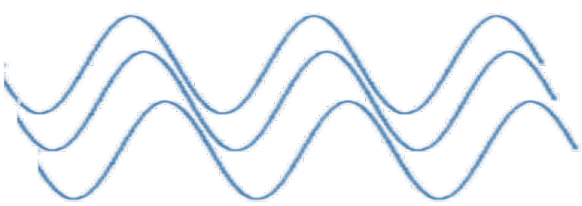
Dr. Steven R. Cranmer
Dr. Thomas E. Berger

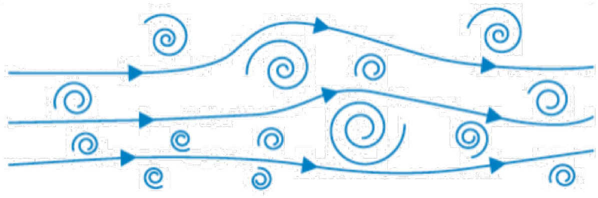
Outline

1. Observations of waves & turbulence
2. Types of fluctuations (MHD, acoustic-gravity, thin-tube modes)
3. How are waves generated ?
4. How do waves propagate & evolve in the corona ?
5. How are waves dissipated (to provide some Q_{heat}) ?
6. Turbulence

(0) *Defining our terms*

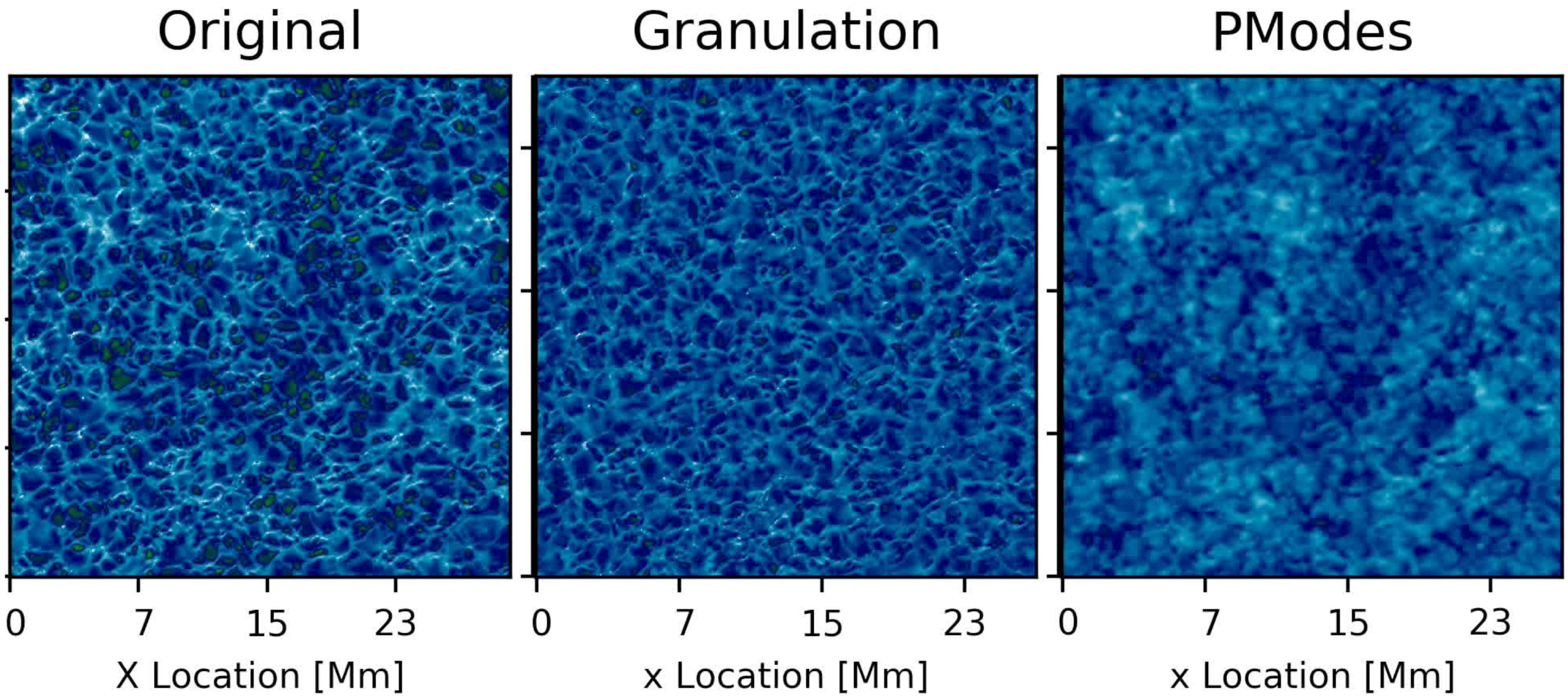
	WAVES	TURBULENCE
Observers	Oscillations (usually with 1 dominant frequency) that propagate through a system. *Also:* a single pulse that propagates through a system ("shock wave")?	Random/stochastic oscillations with unresolved spatial/time scales. (Usually involves a continuous spectrum of frequencies.)
Theorists	Small-amplitude oscillations (i.e., solutions to linearized equations) that propagate through a system.	Stochastic fluctuations that represent a "cascade" of energy across a broad range of spatial/time scales.





(1) *Observations of waves*

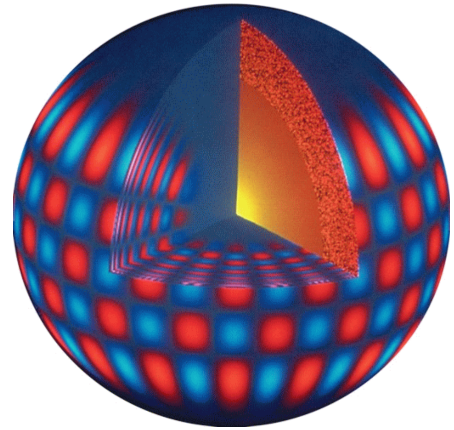
- At the photosphere, we see *both* granulation & the echoes of internally-trapped acoustic-mode waves. McClure et al. (2019, *Solar Phys.*, 294, 18) separated them:

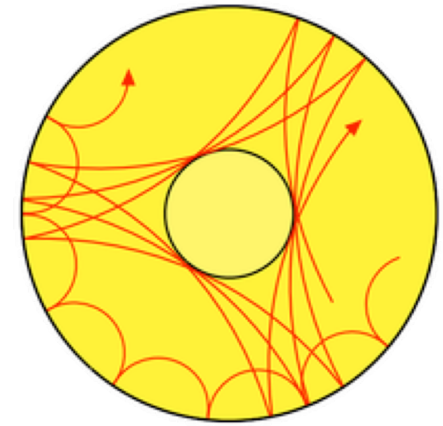


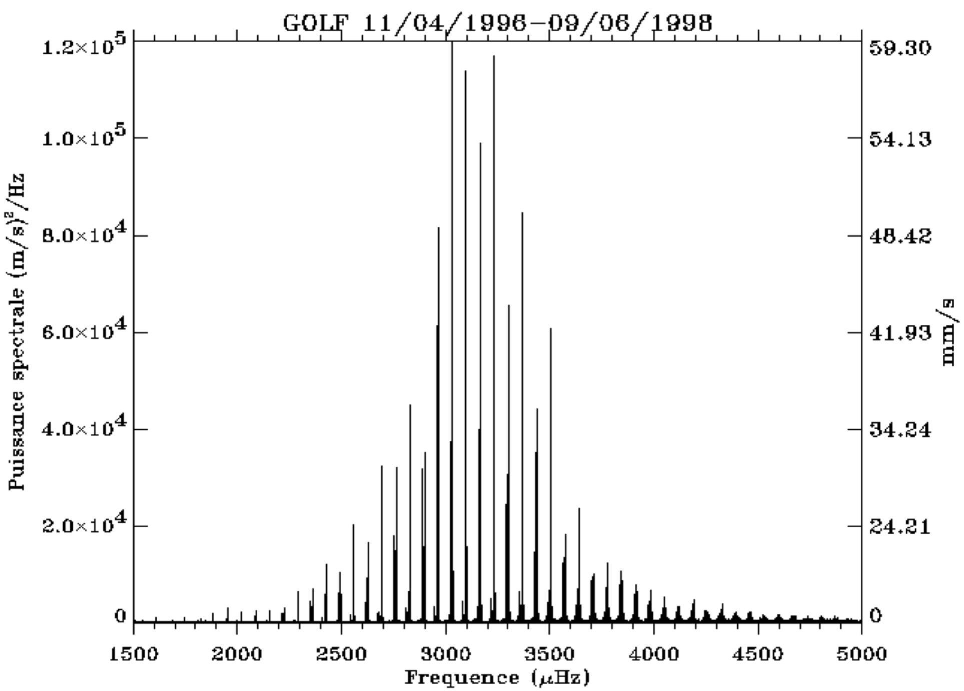
Doppler-shift velocity: black/green (3 km/s up) … white (3 km/s down)

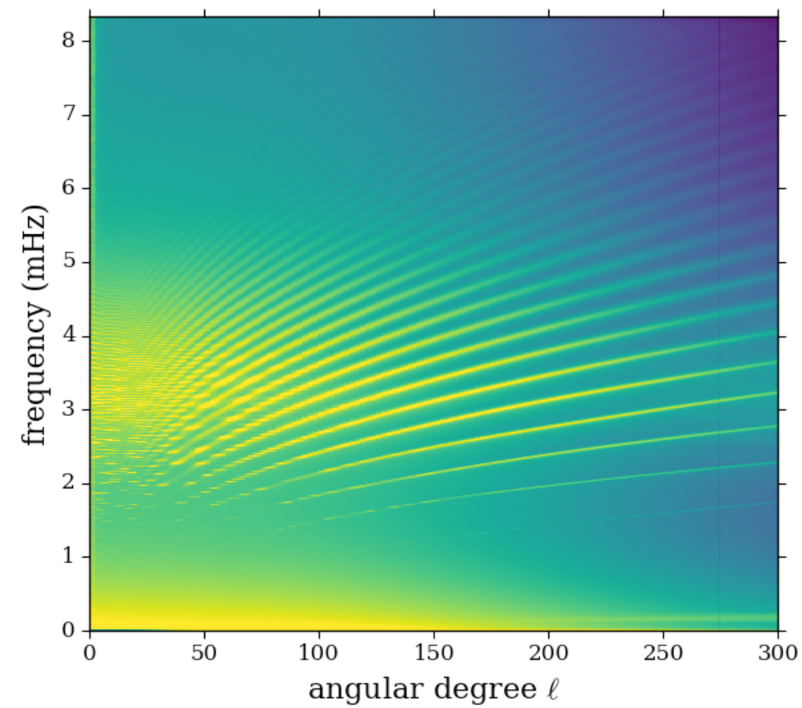
(1) *Observations of waves*

- **Solar interior:** convection excites a broad spectrum of acoustic ("*p*-mode") waves that bounce around.
- Precise frequency measurements allow **helioseismology;** i.e., inference of the (r,θ) dependence of T, c_s, rotation, etc.





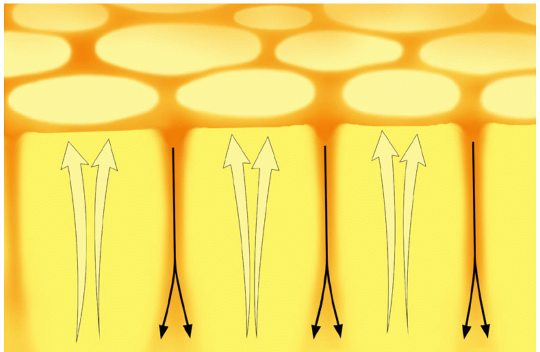


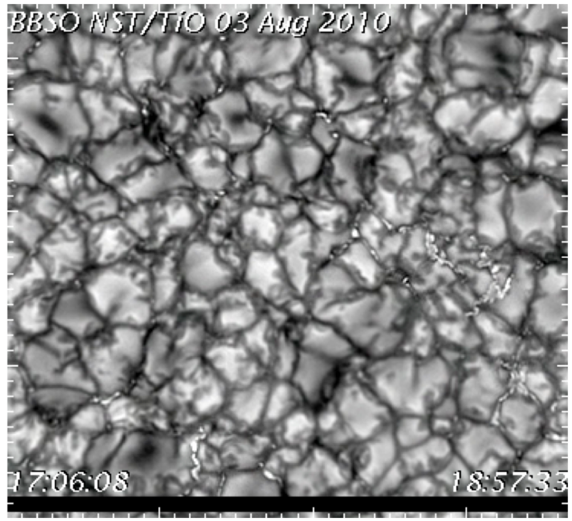


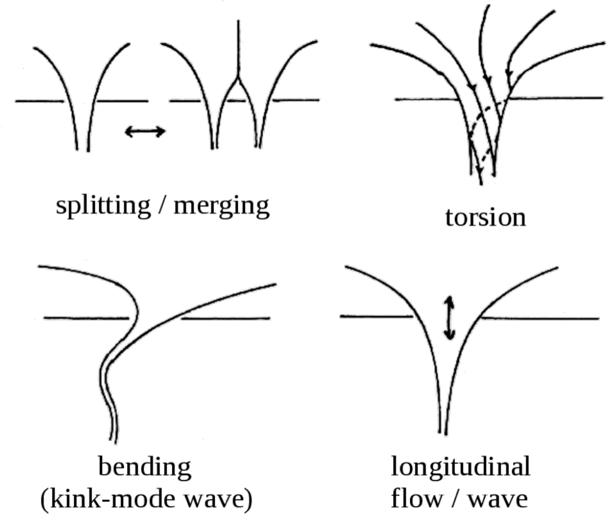
https://www.youtube.com/watch?v=7T0MJDuKNtA

(1) *Observations of waves*

- **Granulation:** the cells themselves aren't waves, but they jostle magnetic flux tubes in the lanes to produce transverse MHD waves higher up…

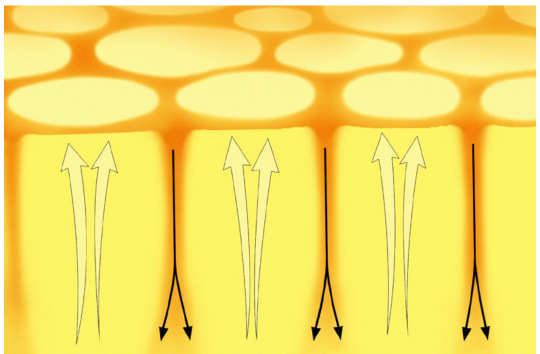


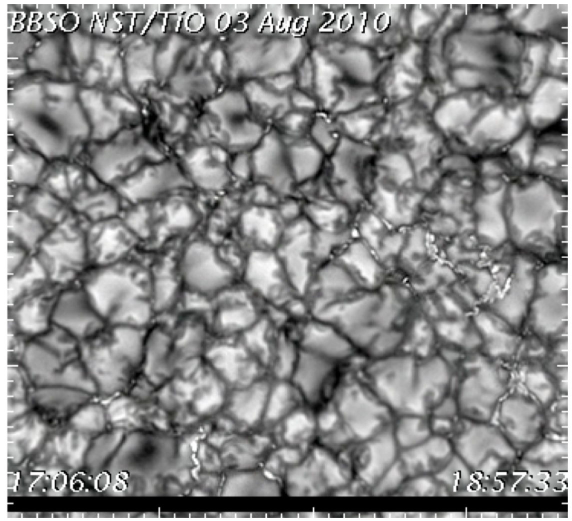


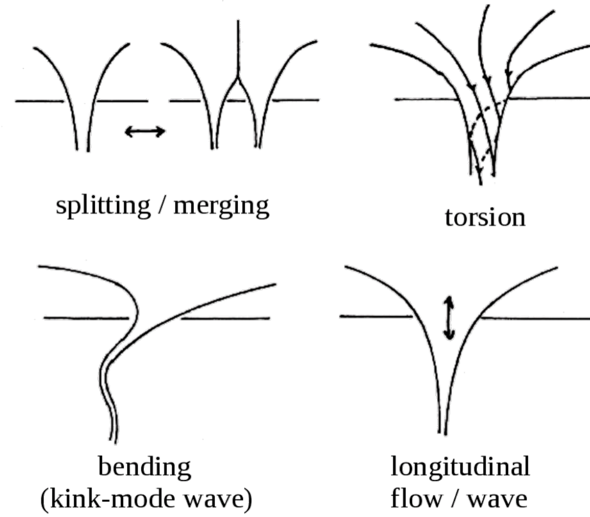


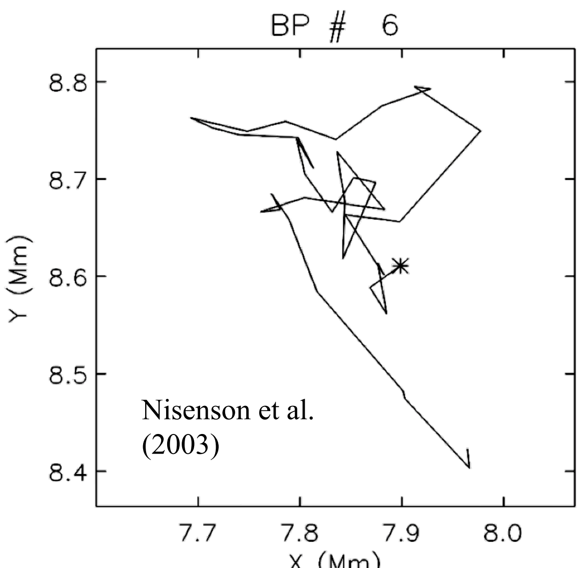
(1) *Observations of waves*

- **Granulation:** the cells themselves aren't waves, but they jostle magnetic flux tubes in the lanes to produce transverse MHD waves higher up…

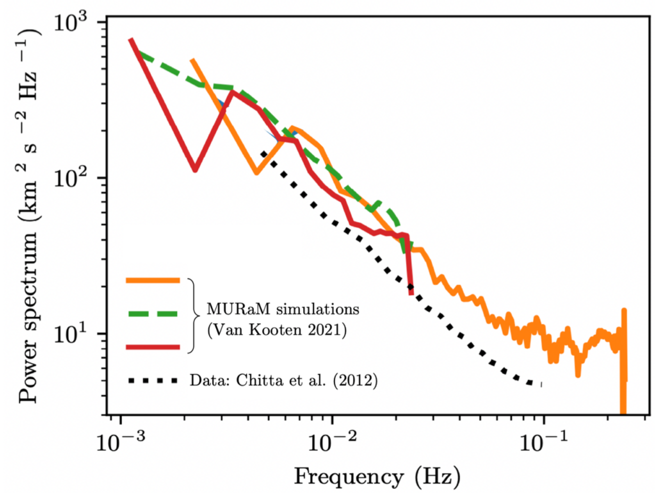




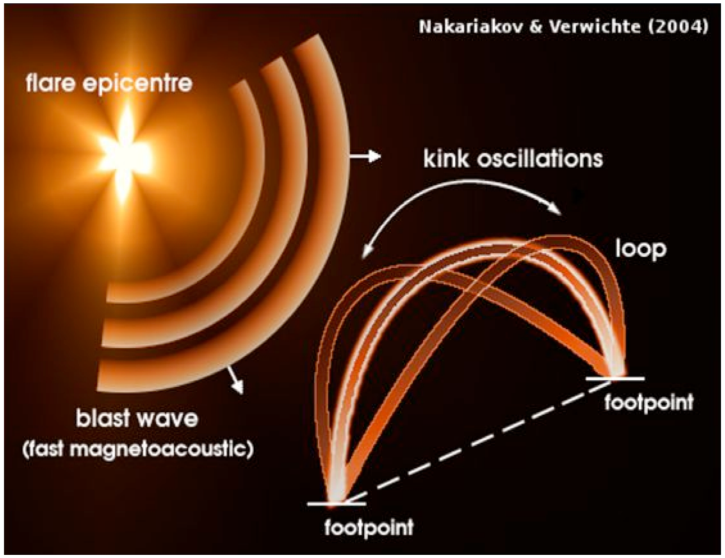


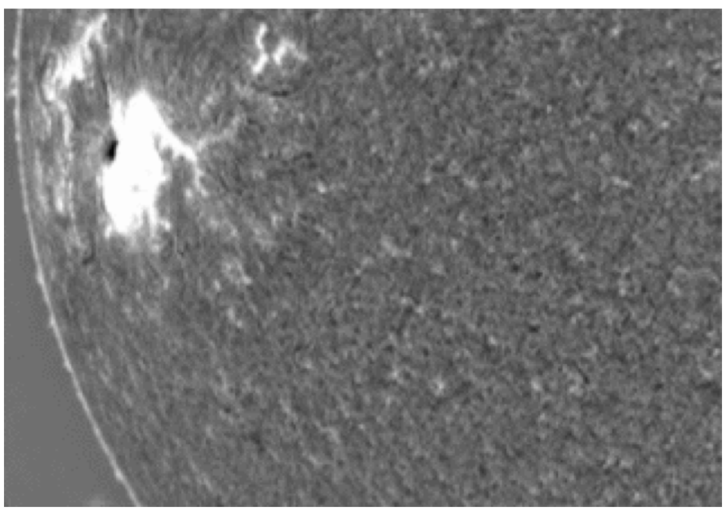


convert
random walk
positions to
velocities



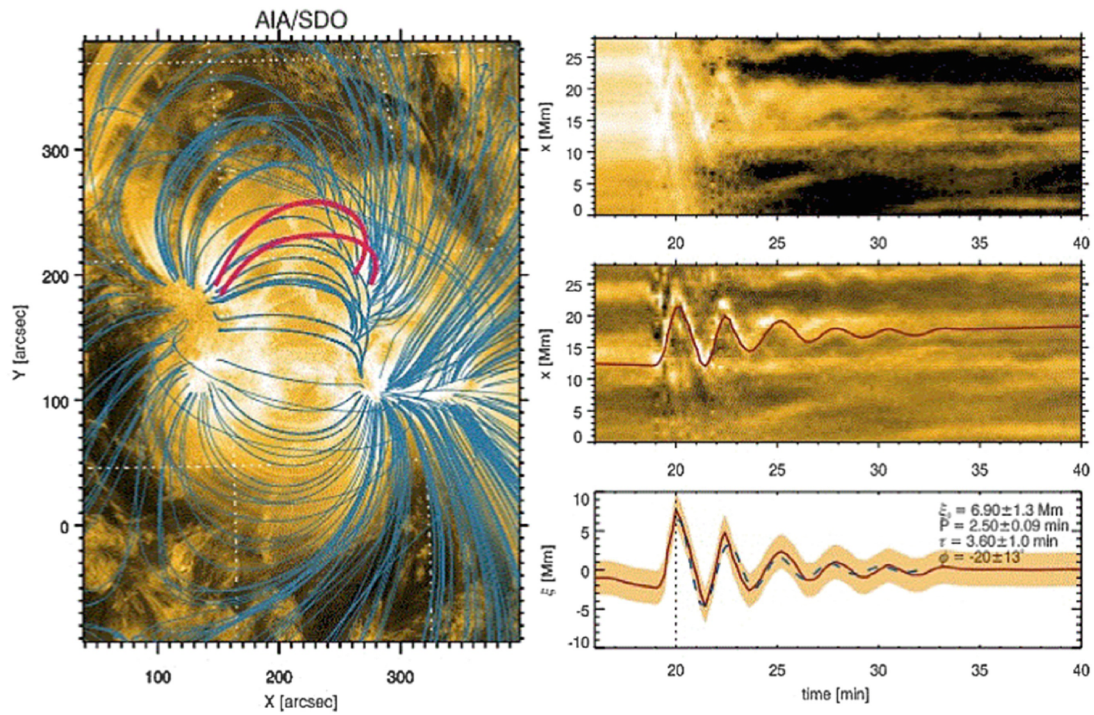
(1) *Observations of waves*





Waves are observed in on-disk coronal images, often as

- single-pulse **blast waves** caused by strong flares ("EUV waves;" "Moreton waves")
- **loop oscillations** that damp rapidly, allowing ***B*** to be inferred ("coronal seismology")



(1) *Observations of waves*

Off-limb coronal wave measurements can be easier to interpret than on-disk…

- Intensity modulations

$$\delta I \;\propto\; (\delta\rho)^{1\text{–}2}$$

- Motion tracking in images

$$\delta x\,,\,\delta y \;\rightarrow\; \delta v_x\,,\,\delta v_y \quad \text{(POS)}$$

- Doppler shifts

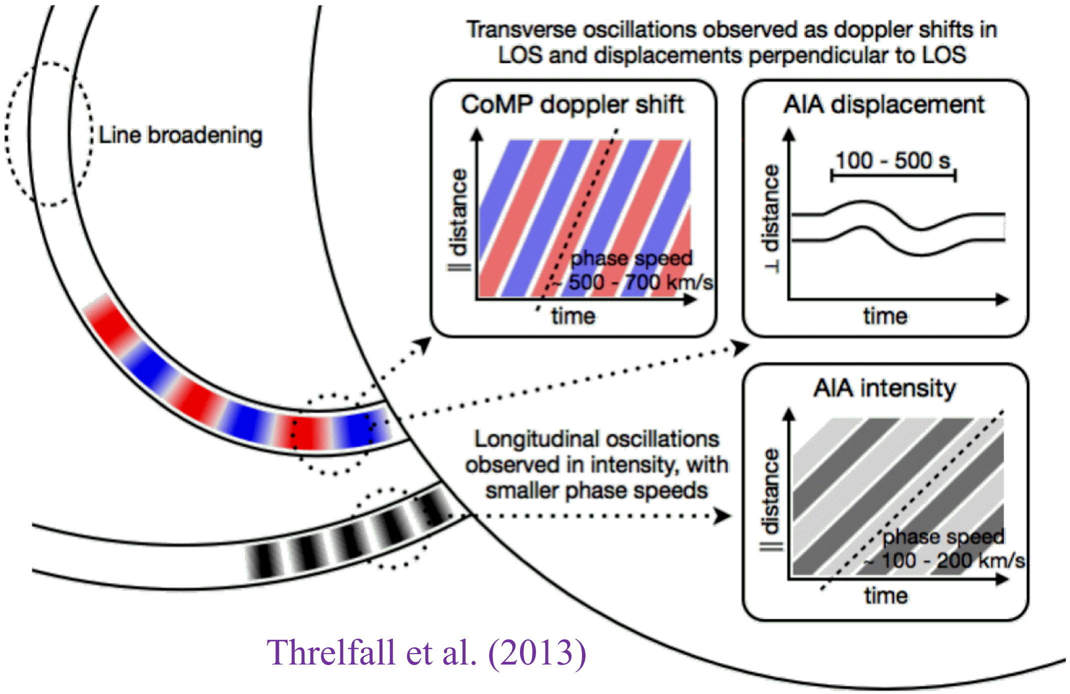
$$\delta\lambda \;\propto\; \delta v_z \quad \text{(LOS)}$$

- Doppler broadening

$$\delta\lambda \;\rightarrow\; \langle \delta v_z \rangle$$

- Radio sounding

$$\delta\tilde{n} \;\rightarrow\; \delta\rho\,,\,\delta B \;\rightarrow\; \delta v$$

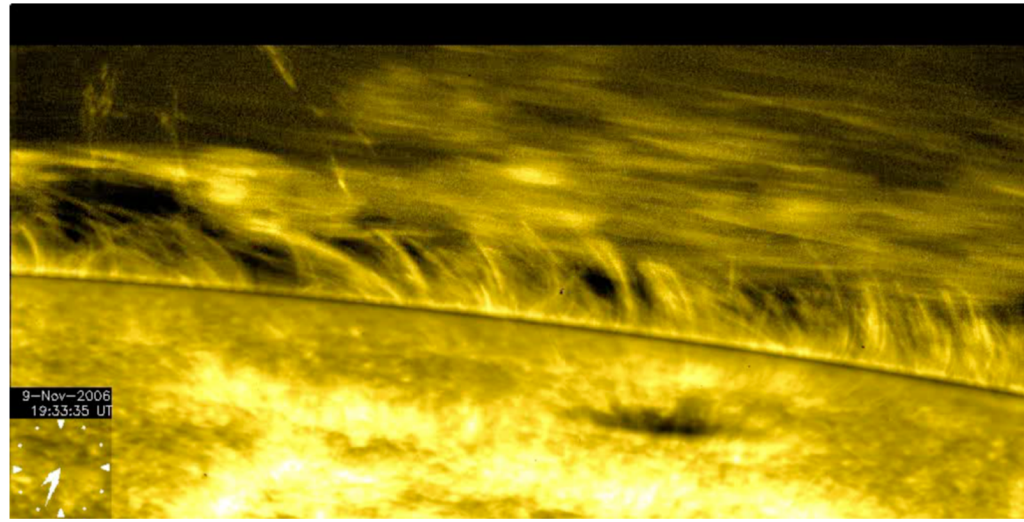


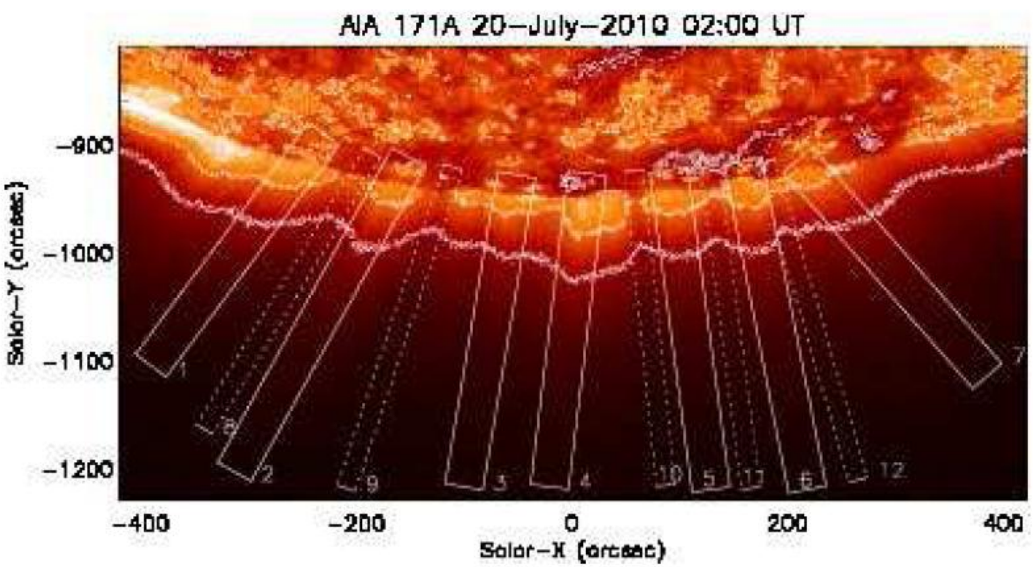
Threlfall et al. (2013)

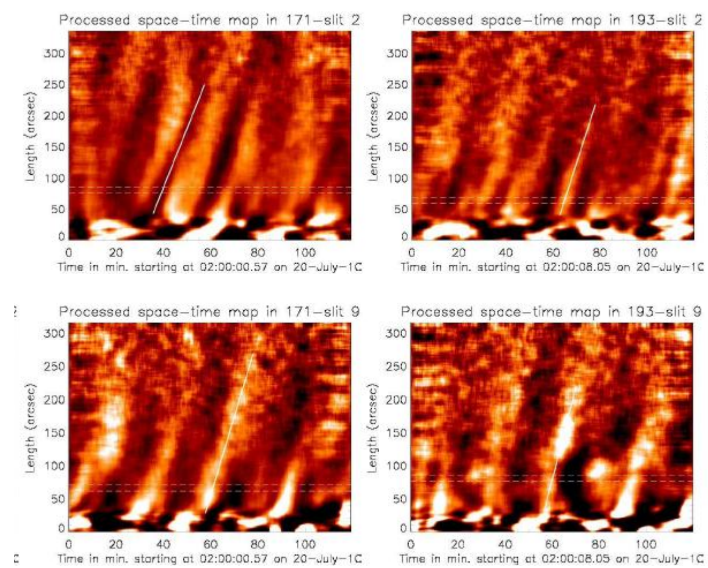
Results: Alfvén-like waves seem to have periods of order 3-5 minutes; compressive waves have periods of order 10-20 minutes.

(1) *Observations of waves*

- Off-limb motion tracking:
  Hinode sees swaying spicules
  (Okamoto et al. 2007).
- Intensity modulations:
  AIA/SDO sees compressive waves
  propagating along polar plumes
  (Krishna Prasad et al. 2011)

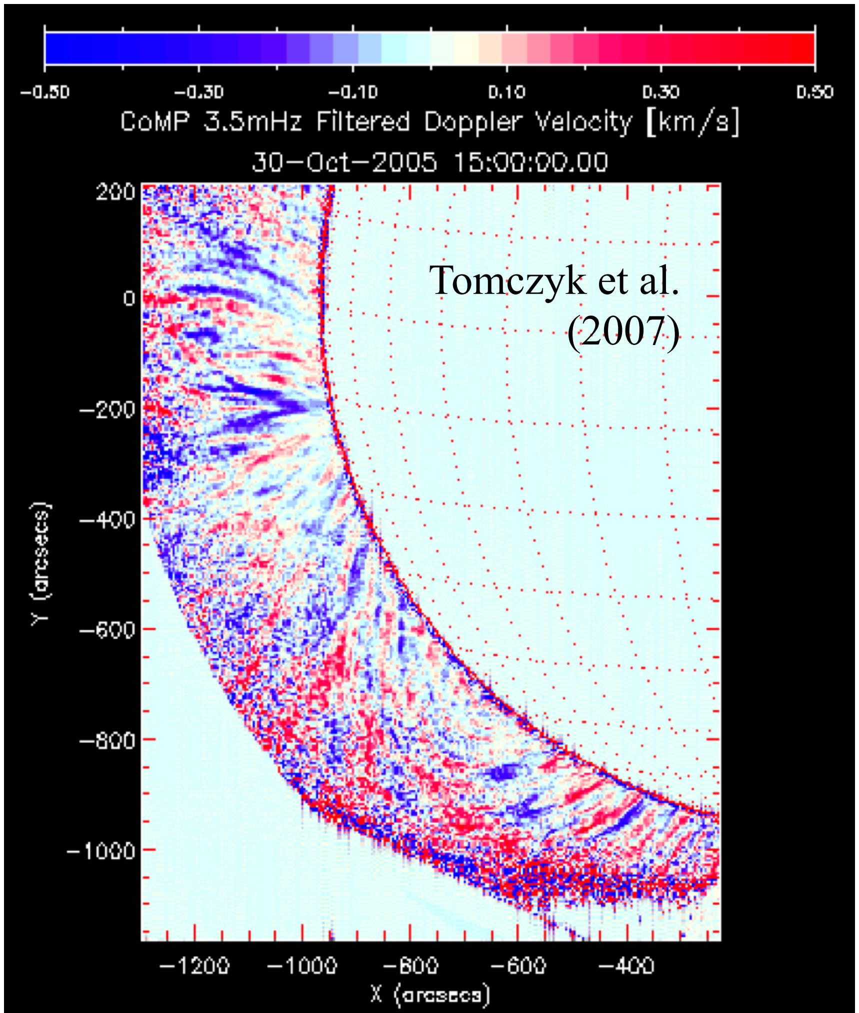




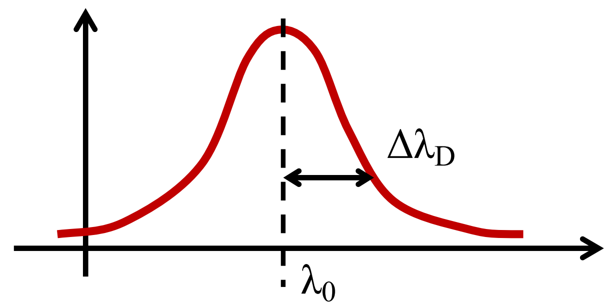


(1) *Observations of waves*

- Off-limb spectroscopy: CoMP sees time-varying Doppler shifts: Alfvén waves and possibly turbulence (e.g., Liu et al. 2014, *ApJ,* 797, 7).



UVCS, SUMER, EIS integrate for times >> 1 wave period to see nonthermal broadening in emission lines…



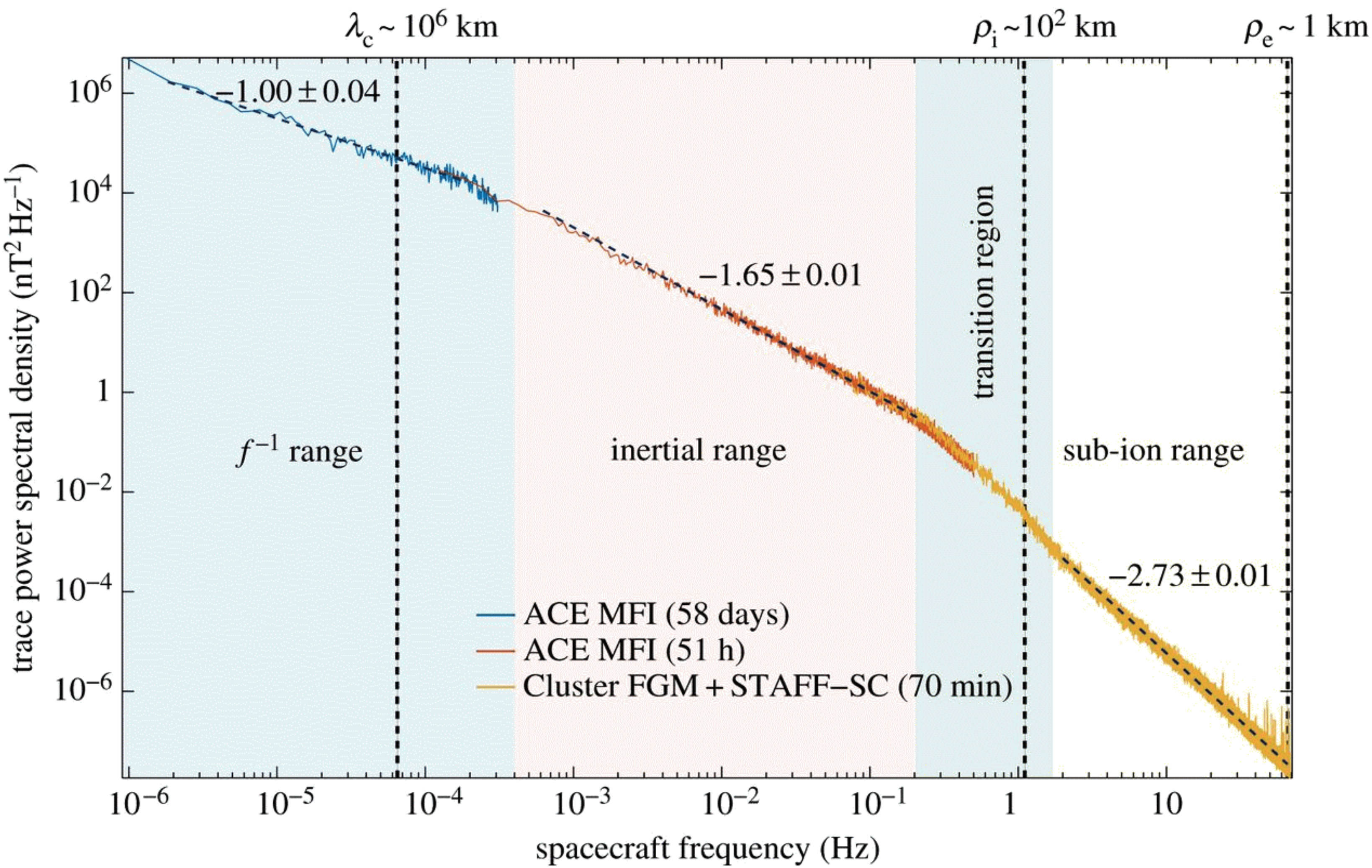
$$\frac{\Delta\lambda_D}{\lambda_0} = \frac{v_{th,eff}}{c} = \frac{1}{c}\sqrt{\frac{2k_B T_{ion}}{m_{ion}} + \xi^2}$$

thermal width

nonthermal width (waves?)

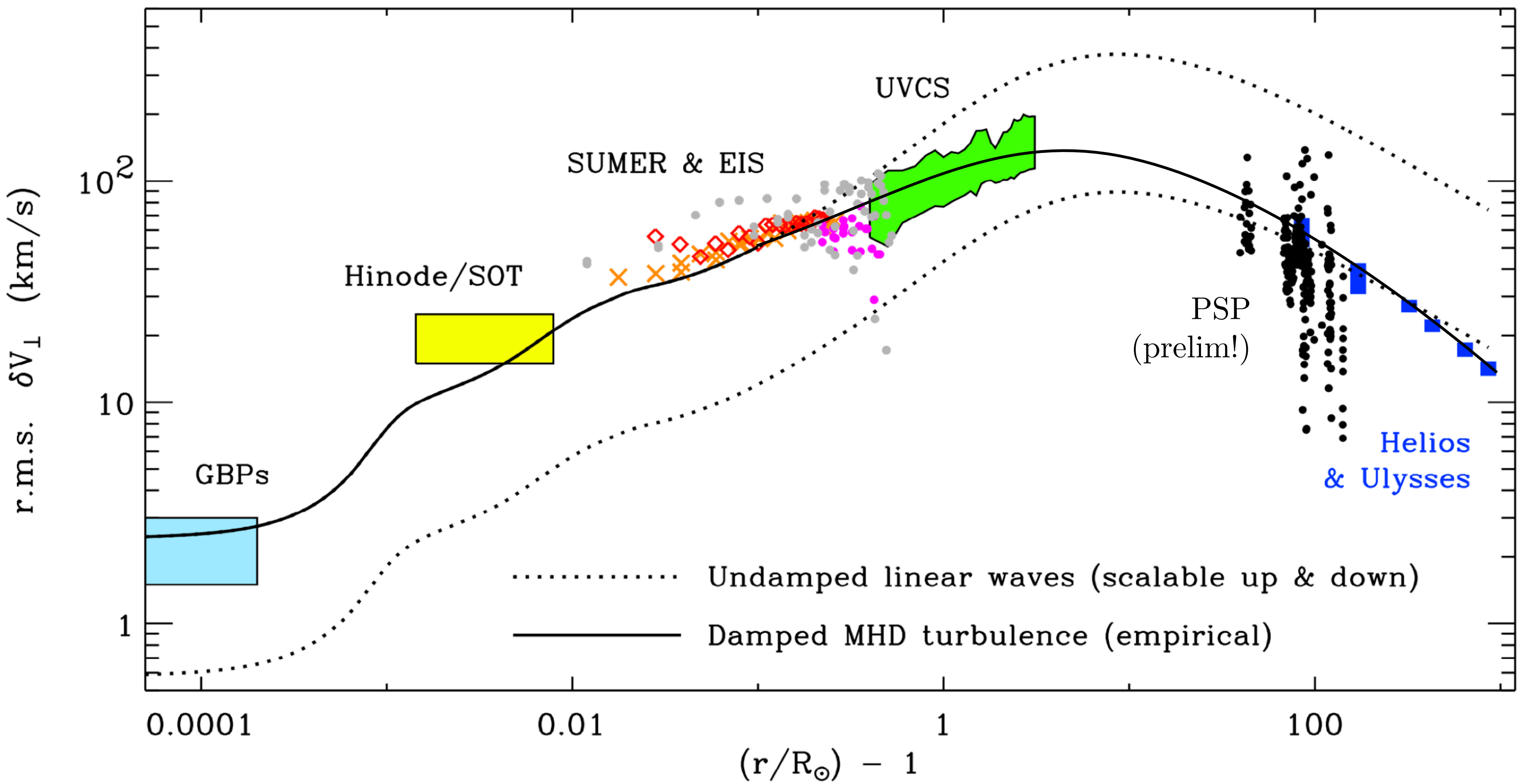
(1) *Observations of waves*

- In the solar wind *in situ* instruments detect broad-band turbulent fluctuations in just about every measurable plasma parameter (e.g., Kiyani et al. 2015):



(1) *Observations of waves*

- Putting it all together (here, for Alfvénic fluctuations in polar coronal holes connected to the fast solar wind), can reveal the global energy budget for waves/turbulence…

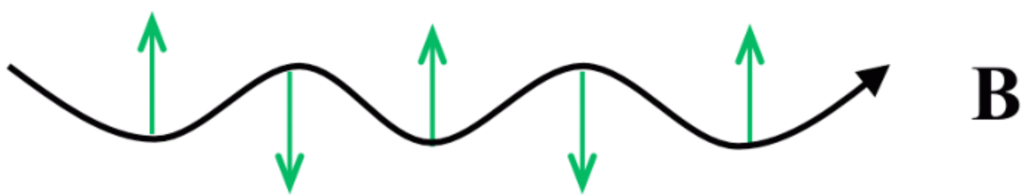


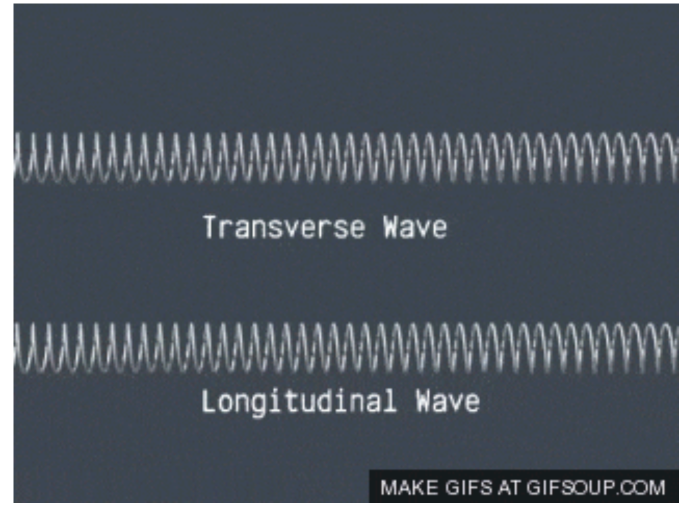
(2) *Types of fluctuations*

I've been talking a lot about two main flavors:

- **Alfvén** (transverse, incompressive, magnetic)

$$U_{\mathrm{A}} = \frac{1}{2}\rho_0(\delta v_\perp)^2 + (\delta B_\perp)^2/8\pi$$





(2) *Types of fluctuations*

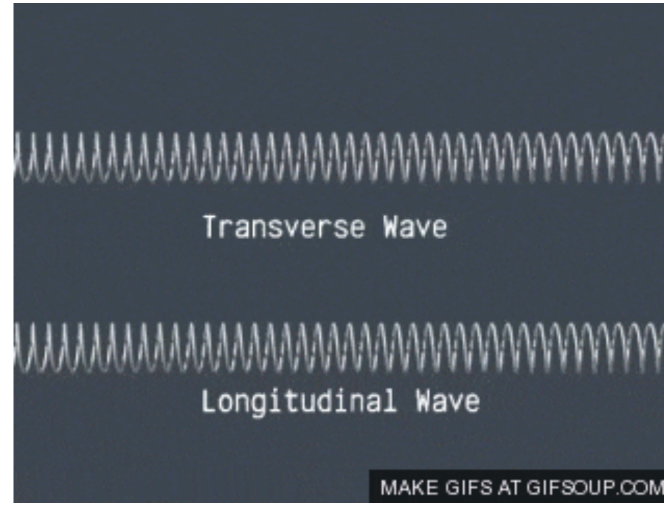
I've been talking a lot about two main flavors:

- **Alfvén** (transverse, incompressive, magnetic)

$$U_{\rm A} = \frac{1}{2}\rho_0(\delta v_\perp)^2 + (\delta B_\perp)^2/8\pi$$

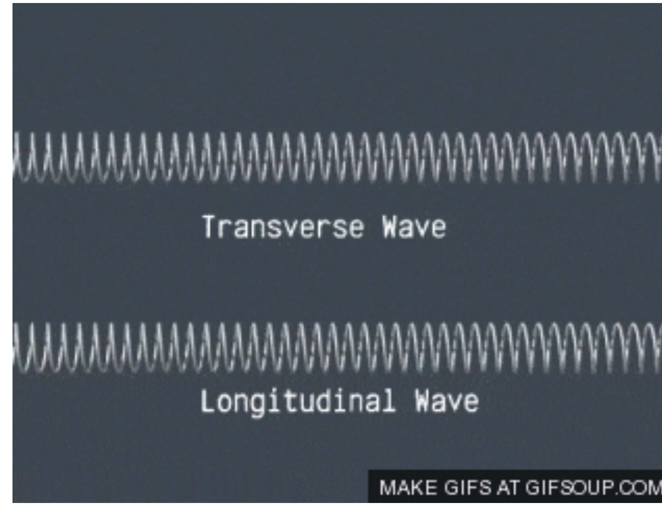
- **Acoustic** (longitudinal, compressive, sound)

$$U_{\rm S} = \frac{1}{2}\rho_0(\delta v_\parallel)^2 + \frac{1}{2}\rho_0\, c_s^2\,(\delta\rho/\rho_0)^2$$



(2) *Types of fluctuations*

I've been talking a lot about two main flavors:



- **Alfvén** (transverse, incompressive, magnetic)

$$U_{\rm A} \ = \ \frac{1}{2}\rho_0(\delta v_\perp)^2 \ + \ (\delta B_\perp)^2/8\pi$$

- **Acoustic** (longitudinal, compressive, sound)

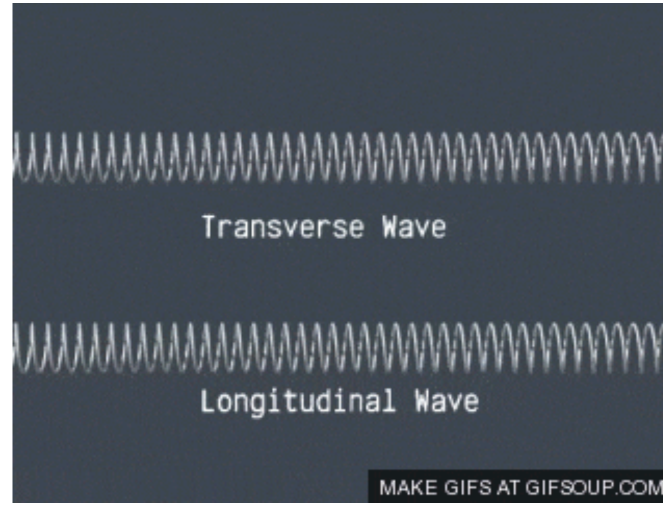
$$U_{\rm S} \ = \ \frac{1}{2}\rho_0(\delta v_\parallel)^2 \ + \ \frac{1}{2}\rho_0 \, c_s^2 \, (\delta\rho/\rho_0)^2$$

Alfvén waves propagate along **B** at $V_{\rm A}$ … acoustic waves propagate at $c_{\rm s} \propto T^{1/2}$

$$\beta \ = \ \frac{P_{\rm gas}}{P_{\rm mag}} \ \approx \ \left(\frac{c_s}{V_{\rm A}}\right)^2$$

(2) *Types of fluctuations*

I've been talking a lot about two main flavors:



- **Alfvén** (transverse, incompressive, magnetic)

$$U_{\mathrm{A}} = \frac{1}{2}\rho_0(\delta v_\perp)^2 + (\delta B_\perp)^2/8\pi$$

- **Acoustic** (longitudinal, compressive, sound)

$$U_{\mathrm{S}} = \frac{1}{2}\rho_0(\delta v_\parallel)^2 + \frac{1}{2}\rho_0\, c_s^2\, (\delta\rho/\rho_0)^2$$

Alfvén waves propagate along **B** at V_{A} … acoustic waves propagate at $c_{\mathrm{s}} \propto T^{1/2}$

In an ideal MHD plasma, there are really 3 modes: Alfven & fast/slow magnetosonic waves (the latter being magnetic modifications of acoustic waves).

Depending on the angle between their wavevector **k** and the background field **B**, fast/slow modes can propagate at phase speeds anywhere between 0 and $(V_{\mathrm{A}}^2 + c_{\mathrm{s}}^2)^{1/2}$ and "carry" all 5 different types of amplitude fluctuations ($\delta v_\perp$, $\delta v_\parallel$, $\delta B_\perp$, $\delta B_\parallel$, $\delta\rho$).

(2) *Types of fluctuations*

- Sometimes one hears solar physicists talking about *p*-modes & *g*-modes above the surface. If we ignore the magnetic field, but take **gravitational stratification** into account, we get new flavors of waves.

(2) *Types of fluctuations*

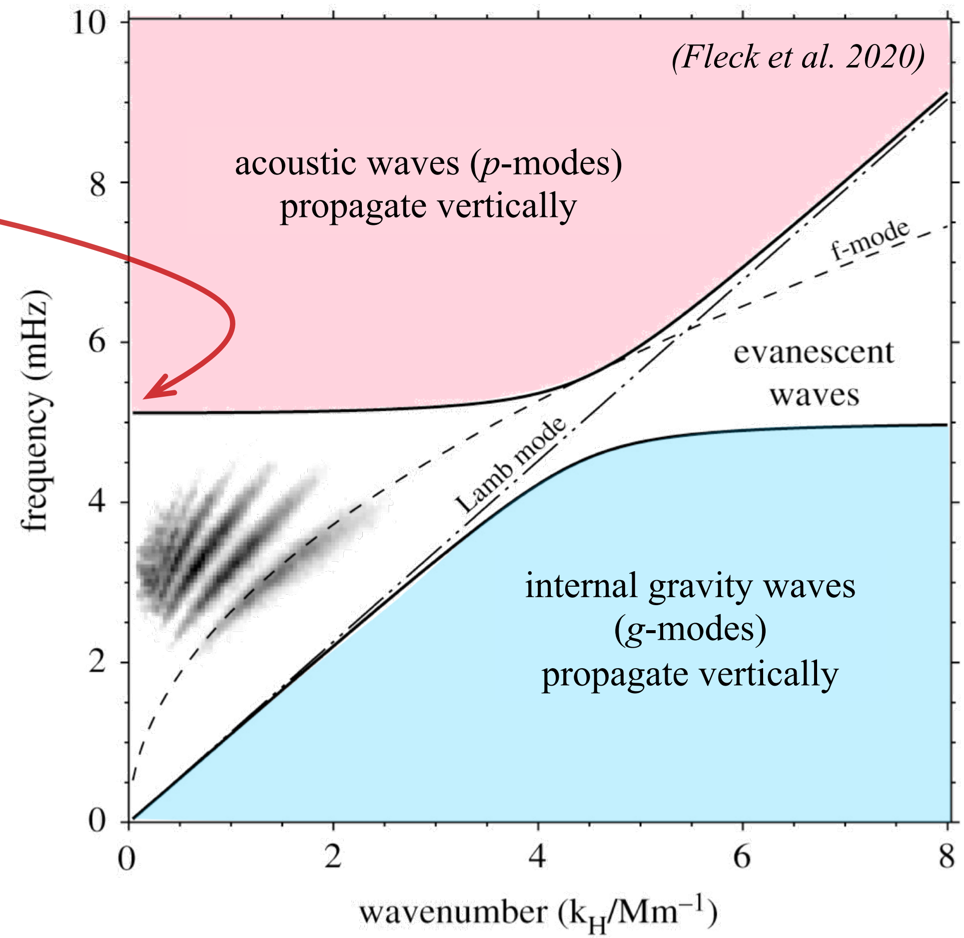
- Sometimes one hears solar physicists talking about *p*-modes & *g*-modes above the surface. If we ignore the magnetic field, but take **gravitational stratification** into account, we get new flavors of waves.

- For vertically propagating sound (*p*-mode) waves, there's an **acoustic cutoff** frequency

$$\omega_{\rm ac} \approx g/c_s$$

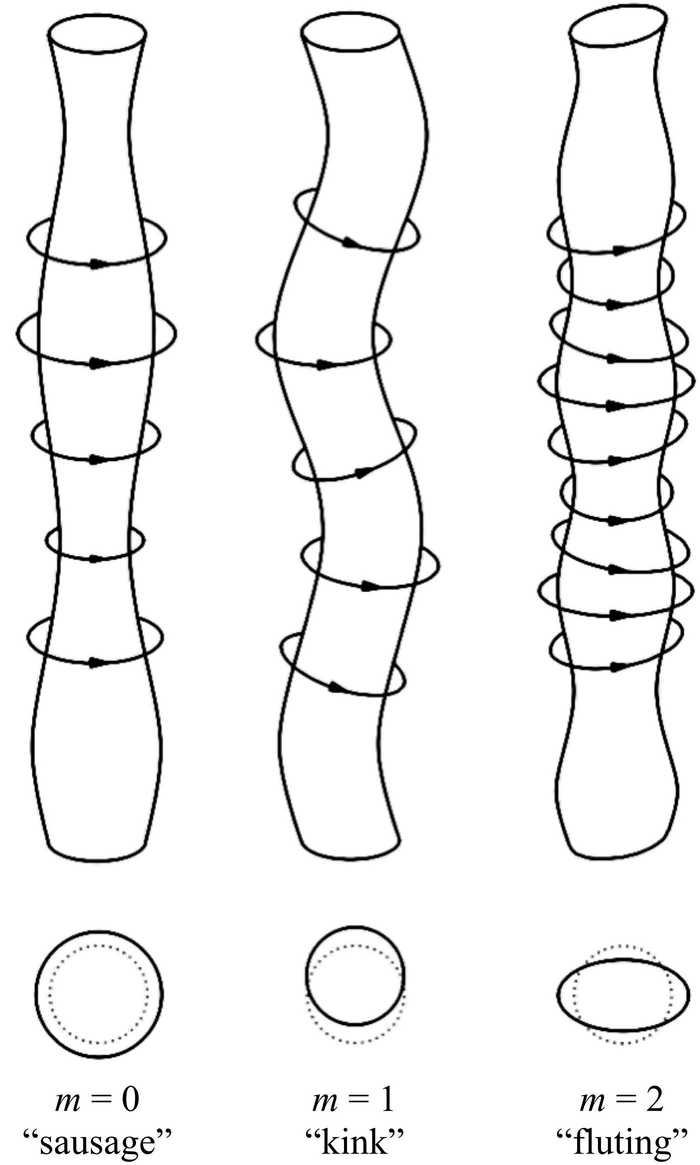
below which waves cannot carry energy up (i.e., they're "standing waves," and their energy density *U* decays evanescently with height).

- There must be strong ***g*-modes** in the deep solar interior, but evidence for them surviving up to the surface is scant…



(2) *Types of fluctuations*

- When the densities & field strengths inside **magnetic flux tubes** ≠ properties outside, there arise even more flavors of wave modes.

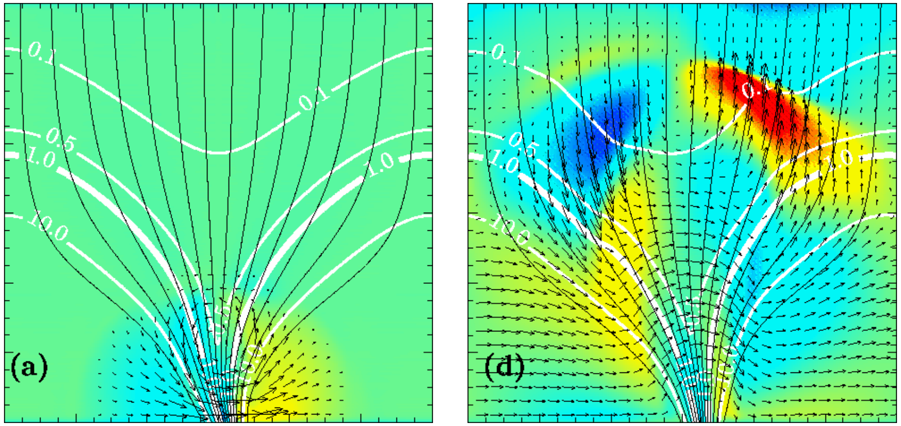


(2) *Types of fluctuations*

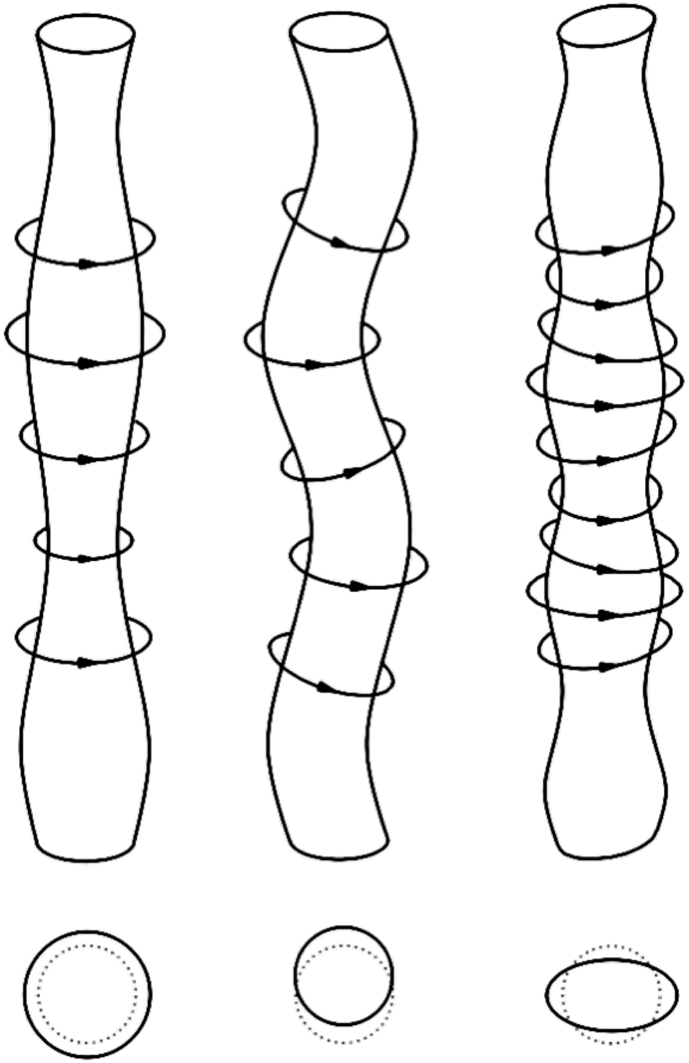
- When the densities & field strengths inside **magnetic flux tubes** $\neq$ properties outside, there arise even more flavors of wave modes.
- Transverse ($m > 0$) modes propagate at a weighted average of inside & outside speeds:

$$V_{\rm ph} = \sqrt{\frac{\rho_i V_{\rm Ai}^2 + \rho_e V_{\rm Ae}^2}{\rho_i + \rho_e}}$$

- In regions where β ~ 1, kink-mode type jostling can excite compressive modes, too…



(Hasan et al. 2005)



(3) *How are waves generated?*

We've mentioned some mechanisms already…

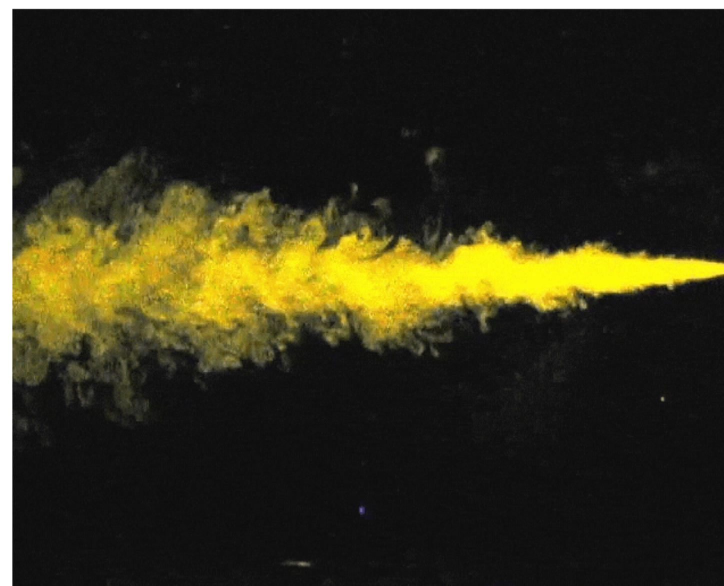
- Vertical flux tubes are jostled, and kink-mode waves propagate up.
- A remnant of convection-driven *p*-mode power survives ("leaks") to large heights.
- Flares (or nanoflares?) trigger pulses that expand out as wavefronts.

(3) *How are waves generated?*

We've mentioned some mechanisms already…

- Vertical flux tubes are jostled, and kink-mode waves propagate up.
- A remnant of convection-driven *p*-mode power survives ("leaks") to large heights.
- Flares (or nanoflares?) trigger pulses that expand out as wavefronts.
- Lighthill's (1952) theory of jet-engine "noise" has been long suspected to be relevant to the generation of waves from incoherent sources like convection…

$$F \approx \frac{\rho\,(\delta v)^3}{\ell_{\rm conv}} \left(\frac{\delta v}{c_s}\right)^5$$

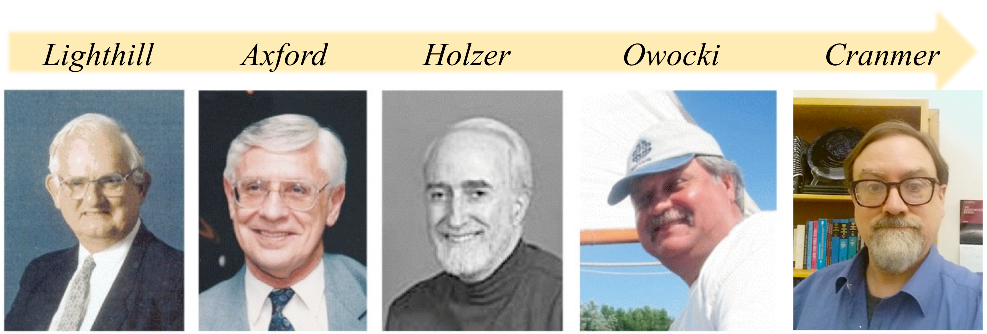


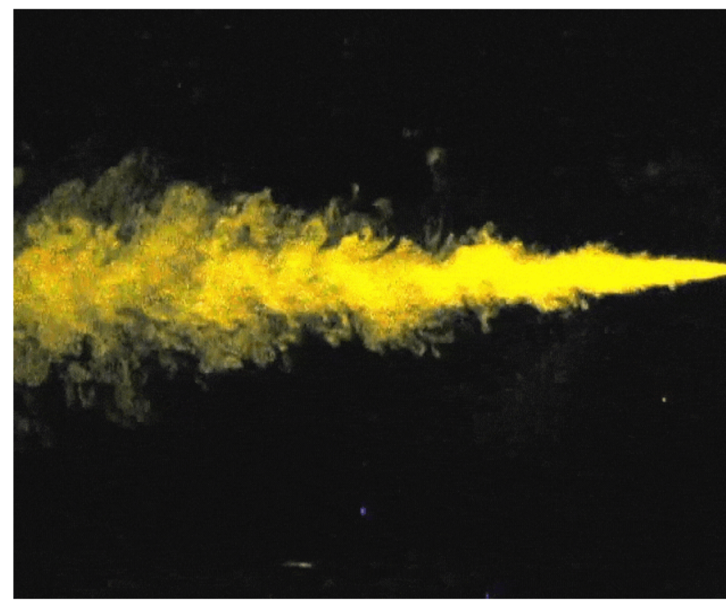
(3) *How are waves generated?*

We've mentioned some mechanisms already…

- Vertical flux tubes are jostled, and kink-mode waves propagate up.
- A remnant of convection-driven *p*-mode power survives ("leaks") to large heights.
- Flares (or nanoflares?) trigger pulses that expand out as wavefronts.
- Lighthill's (1952) theory of jet-engine "noise" has been long suspected to be relevant to the generation of waves from incoherent sources like convection…

$$F \approx \frac{\rho\,(\delta v)^3}{\ell_{\rm conv}} \left(\frac{\delta v}{c_s} \right)^5$$





(4) *How do waves propagate up?*

- Specifically, how do their **amplitudes** ($\delta v_\perp$, $\delta v_\parallel$, $\delta B_\perp$, $\delta B_\parallel$, $\delta\rho$) evolve with height?

(4) *How do waves propagate up?*

- Specifically, how do their **amplitudes** ($\delta v_\perp$, $\delta v_\parallel$, $\delta B_\perp$, $\delta B_\parallel$, $\delta\rho$) evolve with height?
- Linear waves obey a similar-looking energy conservation equation as those we've seen before…

$$\frac{\partial U}{\partial t} + \nabla \cdot \mathbf{F} = -Q_{\rm damp}$$

Set aside $Q_{\rm damp} \approx Q_{\rm heat}$, and assume it's time-steady:

$$\nabla \cdot \mathbf{F} = 0 \quad \Longrightarrow \quad \frac{1}{A(r)} \frac{\partial}{\partial r} [A(r)\, F(r)] = 0$$

$$\text{i.e.,} \qquad F(r) \propto 1/A(r)$$

(4) *How do waves propagate up?*

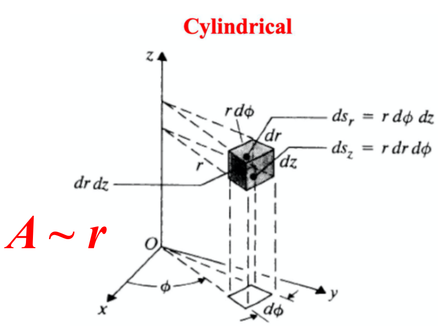
- Specifically, how do their **amplitudes** ($\delta v_\perp$, $\delta v_\parallel$, $\delta B_\perp$, $\delta B_\parallel$, $\delta\rho$) evolve with height?
- Linear waves obey a similar-looking energy conservation equation as those we've seen before…

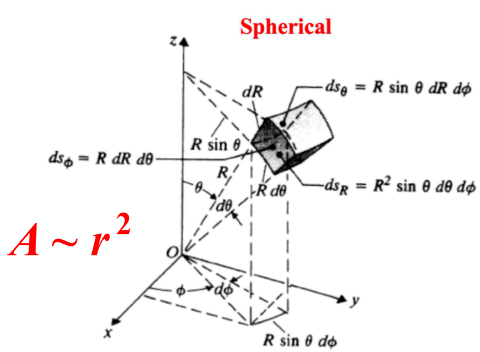
$$\frac{\partial U}{\partial t} + \nabla \cdot \mathbf{F} = -Q_{\rm damp}$$

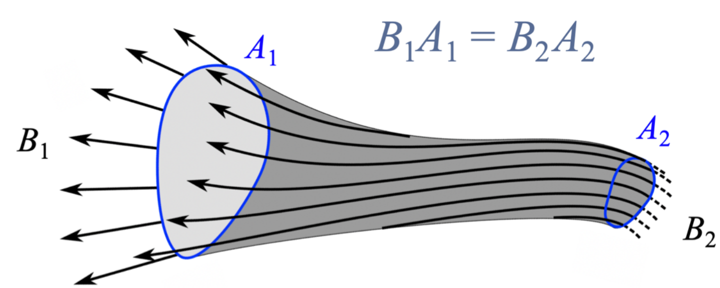
Set aside $Q_{\rm damp} \approx Q_{\rm heat}$, and assume it's time-steady:

$$\nabla \cdot \mathbf{F} = 0 \qquad \Longrightarrow \qquad \frac{1}{A(r)} \frac{\partial}{\partial r} [A(r)\, F(r)] = 0$$

$$\text{i.e.,} \qquad F(r) \propto 1/A(r)$$







(4) *How do waves propagate up?*

- Specifically, how do their **amplitudes** ($\delta v_\perp$, $\delta v_\parallel$, $\delta B_\perp$, $\delta B_\parallel$, $\delta\rho$) evolve with height?
- Linear waves obey a similar-looking energy conservation equation as those we've seen before…

$$\frac{\partial U}{\partial t} + \nabla \cdot \mathbf{F} = -Q_{\rm damp}$$

Set aside $Q_{\rm damp} \approx Q_{\rm heat}$, and assume it's time-steady:

$$\nabla \cdot \mathbf{F} = 0 \qquad \Longrightarrow \qquad \frac{1}{A(r)} \frac{\partial}{\partial r} \left[A(r)\, F(r) \right] = 0$$

$$\text{i.e.,} \qquad F(r) \propto 1/A(r)$$

For the two main types,

$$F = U V_{\rm gr} \approx \begin{cases} \rho_0 (\delta v_\parallel)^2 \, c_s \,, & \text{(sound)} \\ \rho_0 (\delta v_\perp)^2 \, V_{\rm A} \,, & \text{(Alfvén)} \end{cases}$$

(4) *How do waves propagate up?*

- **Sound wave evolution:** assume $A \approx$ constant (i.e., waves just go straight up)

$$\rho_0 (\delta v_\parallel)^2 c_s \approx \text{constant} \quad \Longrightarrow \quad \delta v_\parallel \propto \rho_0^{-1/2} T^{-1/4}$$

For a chromosphere with a "flat" (isothermal) temperature, ignore the $T^{-1/4}$ factor.

For last week's chromosphere model (with $\Lambda \propto T^8$), we get $\delta v_\parallel \propto \rho_0^{-0.438}$

(4) *How do waves propagate up?*

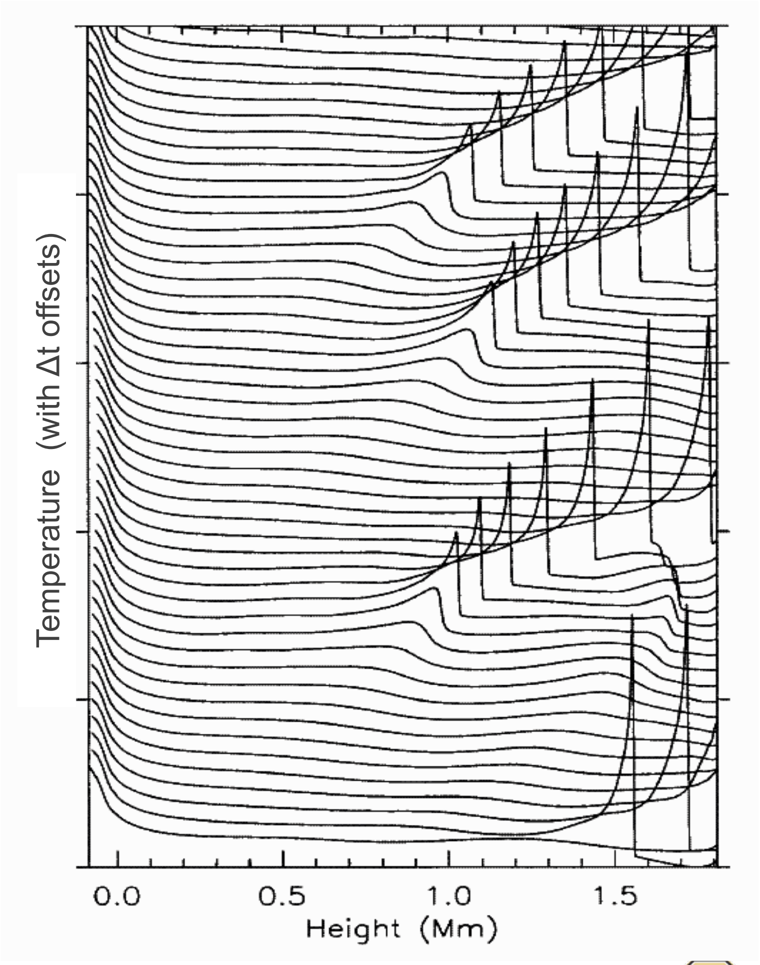
- **Sound wave evolution:** assume $A \approx$ constant (i.e., waves just go straight up)

$$\rho_0 (\delta v_{\|})^2 \, c_s \;\approx\; \text{constant} \qquad \Longrightarrow \qquad \delta v_{\|} \;\propto\; \rho_0^{-1/2} \, T^{-1/4}$$

For a chromosphere with a "flat" (isothermal) temperature, ignore the $T^{-1/4}$ factor.

For last week's chromosphere model (with $\Lambda \propto T^8$), we get $\delta v_{\|} \propto \rho_0^{-0.438}$

- Thus, sound waves grow in amplitude with increasing height… until the point where $\delta v_{\|}$ reaches the sound speed c_s .
- Then, because "crests" can easily outpace "troughs," **the waves nonlinearly steepen into shocks.**
- Piston-driven 1D models (e.g., Carlsson & Stein) give rise to intermittent sawtooths…



(4) *How do waves propagate up?*

- **Alfvén wave evolution:** assume $A \approx 1/B$ (i.e., waves follow the field lines)

$$\rho_0 (\delta v_\perp)^2 \, V_\mathrm{A} \sim \rho_0 (\delta v_\perp)^2 \frac{B}{\sqrt{\rho_0}} \propto \frac{1}{A} \sim B \quad \Longrightarrow \quad \delta v_\perp \propto \rho_0^{-1/4}$$

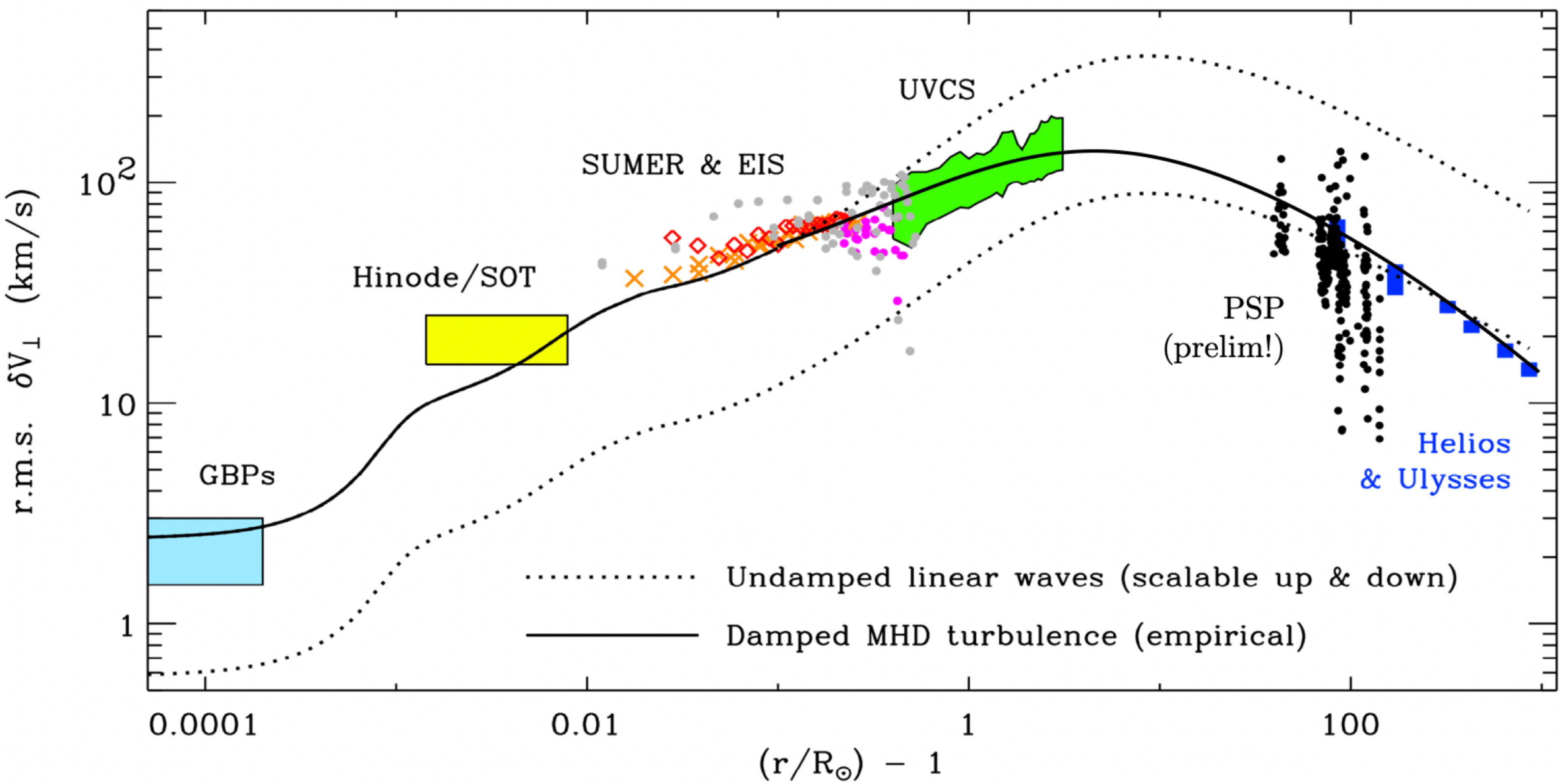
Alfvén waves also grow in amplitude as height increases…

(4) *How do waves propagate up?*

- **Alfvén wave evolution:** assume $A \approx 1/B$ (i.e., waves follow the field lines)

$$\rho_0 (\delta v_\perp)^2 \, V_A \sim \rho_0 (\delta v_\perp)^2 \frac{B}{\sqrt{\rho_0}} \propto \frac{1}{A} \sim B \quad \Longrightarrow \quad \delta v_\perp \propto \rho_0^{-1/4}$$

Alfvén waves also grow in amplitude as height increases… recall obs. potpourri:



(5) *How do waves dissipate → heat ?*

Four general sets of mechanisms:

5a. Collisions (particle-particle interactions)

5b. Entropy increase across shocks (*maybe* a subset of 5a)

5c. Radiation (important in photosphere & low chromosphere)

5d. Collisionless effects (wave-particle interactions)

(5) *How do waves dissipate → heat ?*

Four general sets of mechanisms:

5a. Collisions (particle-particle interactions)

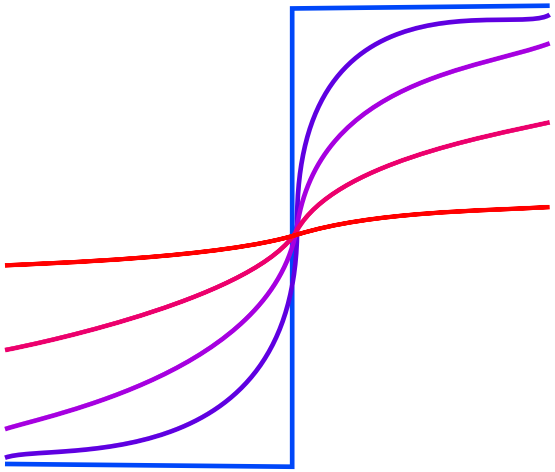
5b. Entropy increase across shocks (*maybe* a subset of 5a)

5c. Radiation (important in photosphere & low chromosphere)

5d. Collisionless effects (wave-particle interactions)

- Collisions occur ~randomly on micro-scales, and they usually add **diffusion terms** to the momentum & energy conservation equations:

viscosity			velocity shear
electrical resistivity	reduces		ion-electron drift (**J**)
heat conductivity			temperature gradient
ambipolar friction			ion-neutral drift



(5) *How do waves dissipate → heat ?*

Collisions produce imaginary terms in the dispersion relation...

$$\omega = kV_{\rm ph} = \omega_r + i\gamma \qquad \{\text{amplitudes}\} \propto e^{i\omega_r t}\, e^{-\gamma t} \qquad Q_{\rm damp} = 2\gamma U$$

(5) *How do waves dissipate → heat ?*

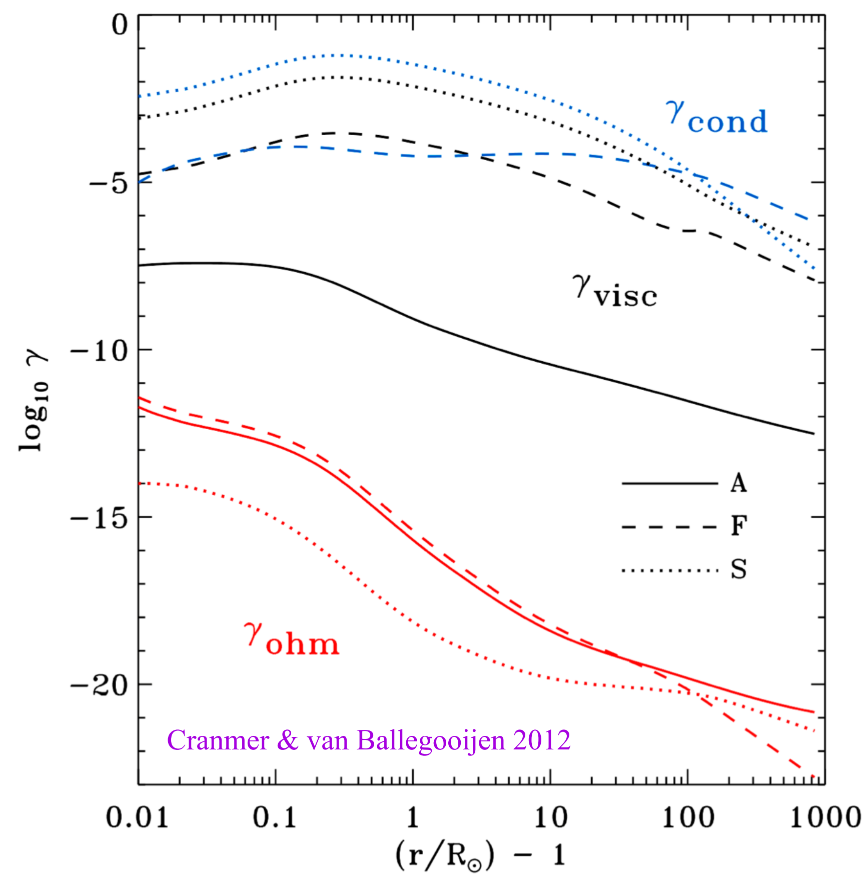
Collisions produce imaginary terms in the dispersion relation...

$$\omega = kV_{\rm ph} = \omega_r + i\gamma \qquad \{\text{amplitudes}\} \propto e^{i\omega_r t}\, e^{-\gamma t} \qquad Q_{\rm damp} = 2\gamma U$$

In the corona & solar wind:

- Compressive (fast & slow) waves are damped most strongly by conduction, over very short length scales:

$$L_{\rm damp} = V_{\rm ph}/\gamma \sim (10^{-4}\ R_{\odot}) \left(\frac{\text{period}}{5\ \text{min}}\right)^2$$



(5) *How do waves dissipate → heat ?*

Collisions produce imaginary terms in the dispersion relation...

$$\omega = kV_{\rm ph} = \omega_r + i\gamma \qquad \{\text{amplitudes}\} \propto e^{i\omega_r t}\, e^{-\gamma t} \qquad Q_{\rm damp} = 2\gamma U$$

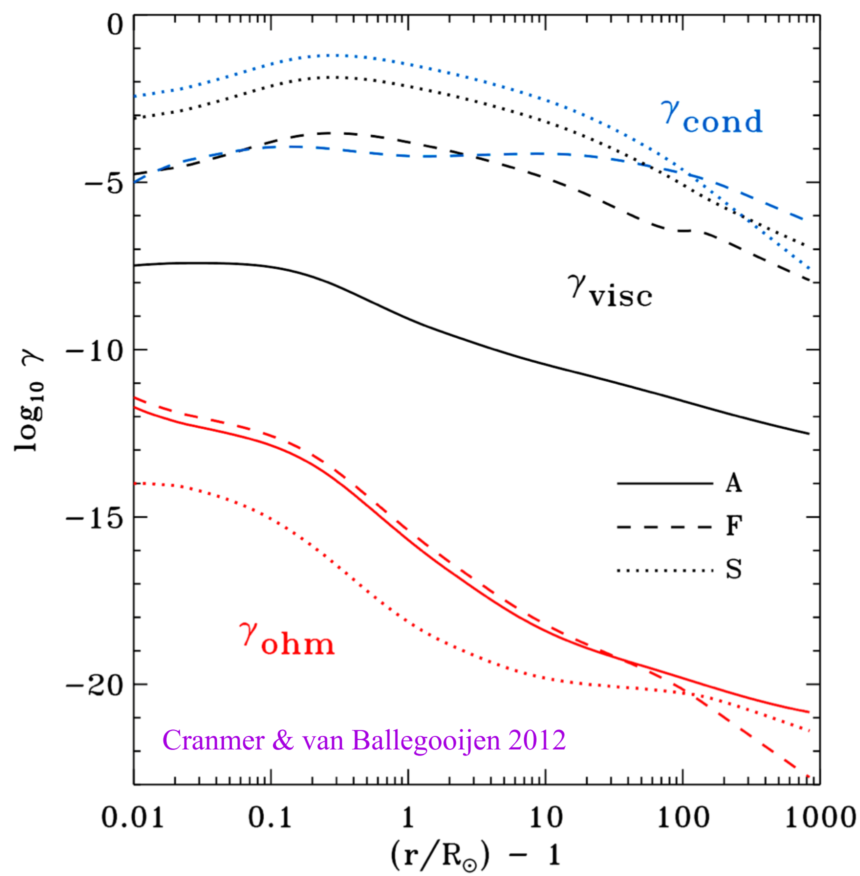
In the corona & solar wind:

- Compressive (fast & slow) waves are damped most strongly by conduction, over very short length scales:

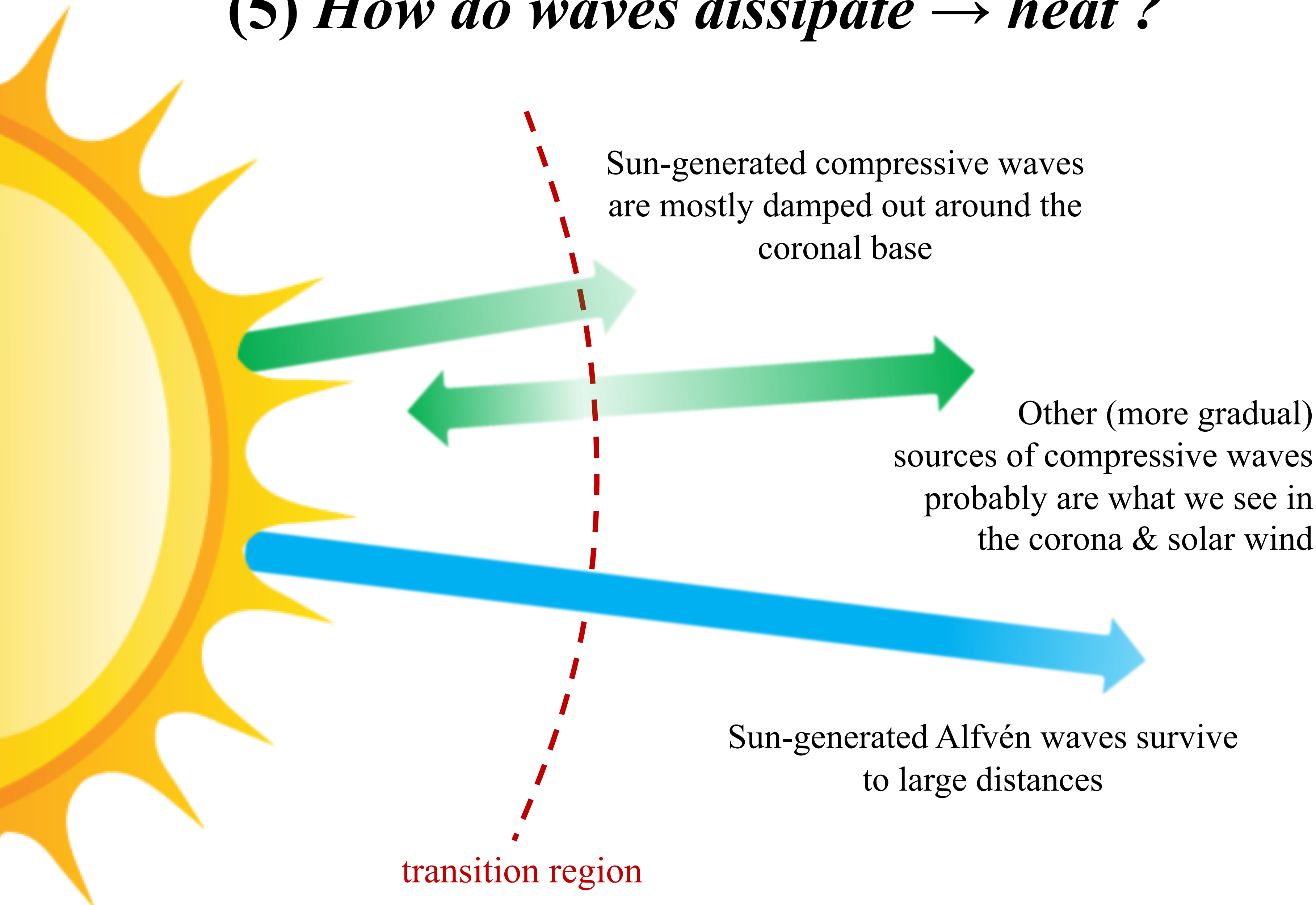
$$L_{\rm damp} = V_{\rm ph}/\gamma \sim (10^{-4}\ R_\odot) \left(\frac{\text{period}}{5\ \text{min}}\right)^2$$

- Alfvén waves don't feel conductive damping, and visc/Ohmic damping is weaker:

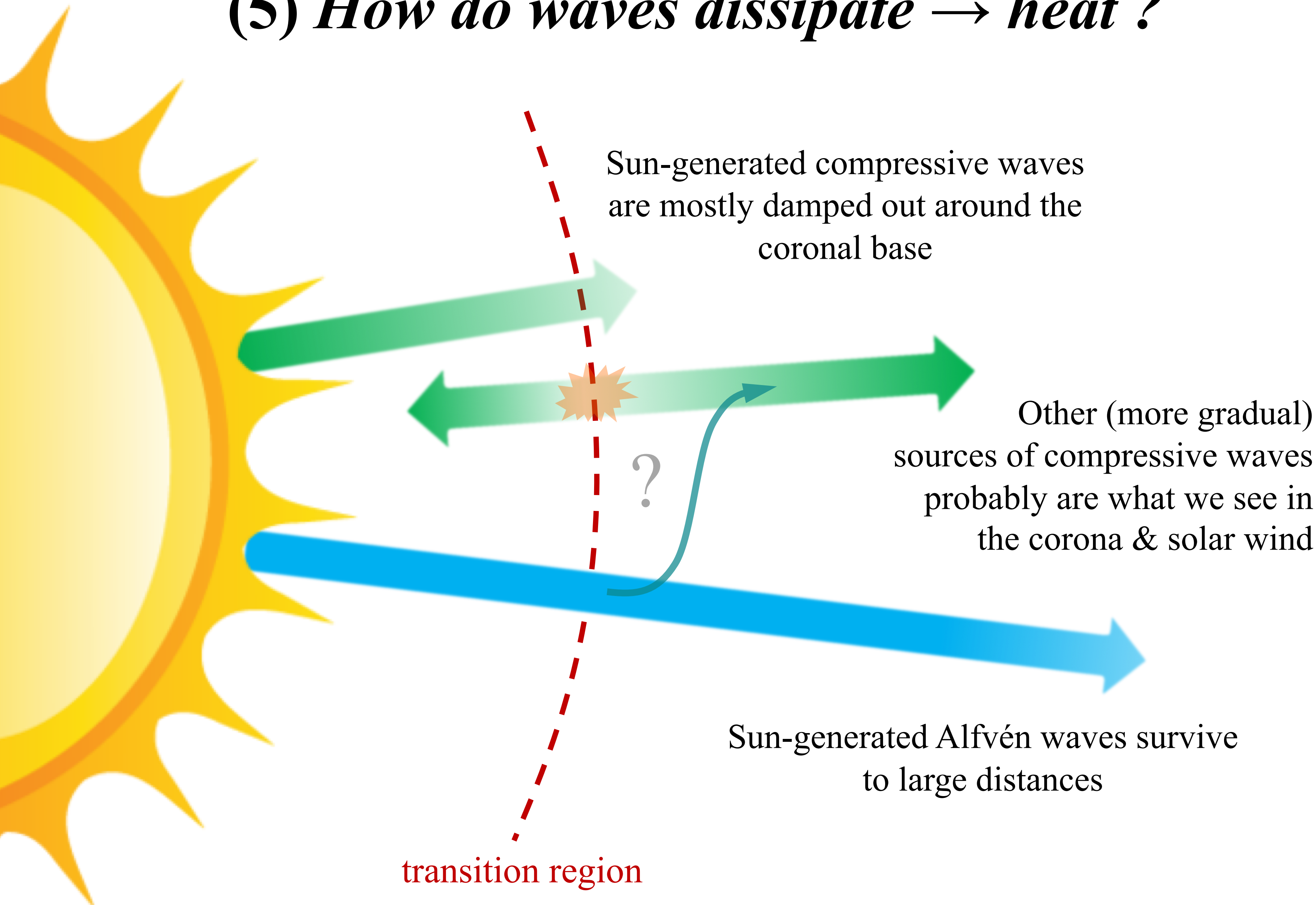
$$L_{\rm damp} = V_{\rm ph}/\gamma \sim (5\ R_\odot) \left(\frac{\text{period}}{5\ \text{min}}\right)^2$$



(5) *How do waves dissipate → heat ?*



(5) *How do waves dissipate → heat ?*

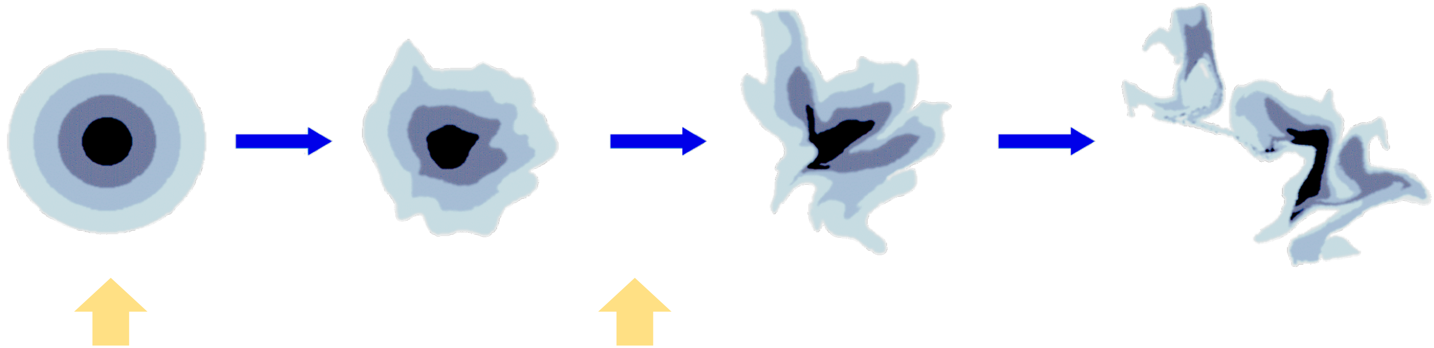


(6) *Turbulence*

- Leonardo da Vinci was probably the first to subject chaotic, nonlinear fluid flows (i.e., *turbolenza*) to scientific scrutiny.
- In the 20th century, Kolmogorov, von Karman, Taylor, Richardson, et al. studied it statistically.

(6) *Turbulence*

- Leonardo da Vinci was probably the first to subject chaotic, nonlinear fluid flows (i.e., *turbolenza*) to scientific scrutiny.
- In the 20th century, Kolmogorov, von Karman, Taylor, Richardson, et al. studied it statistically.
- For hydrodynamic fluids, the nonlinear terms in the momentum conservation equation allow for large-scale eddies to be "shredded…"



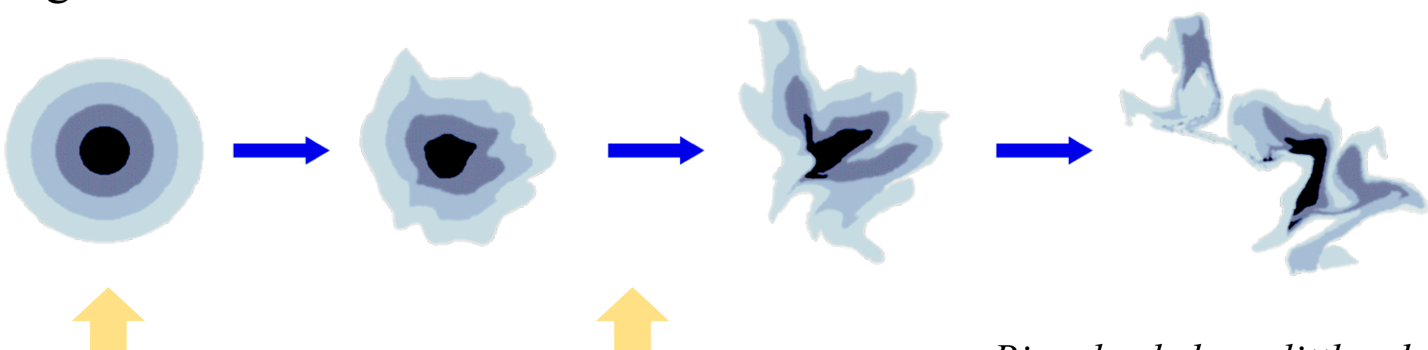
Cartoon: Maron & Goldreich (2001)

"Power" at largest scales (smallest *k*)

"Power" spreads to fill smaller scales (larger *k*)

(6) *Turbulence*

- Leonardo da Vinci was probably the first to subject chaotic, nonlinear fluid flows (i.e., *turbolenza*) to scientific scrutiny.
- In the 20th century, Kolmogorov, von Karman, Taylor, Richardson, et al. studied it statistically.
- For hydrodynamic fluids, the nonlinear terms in the momentum conservation equation allow for large-scale eddies to be "shredded…"



Cartoon: Maron & Goldreich (2001)

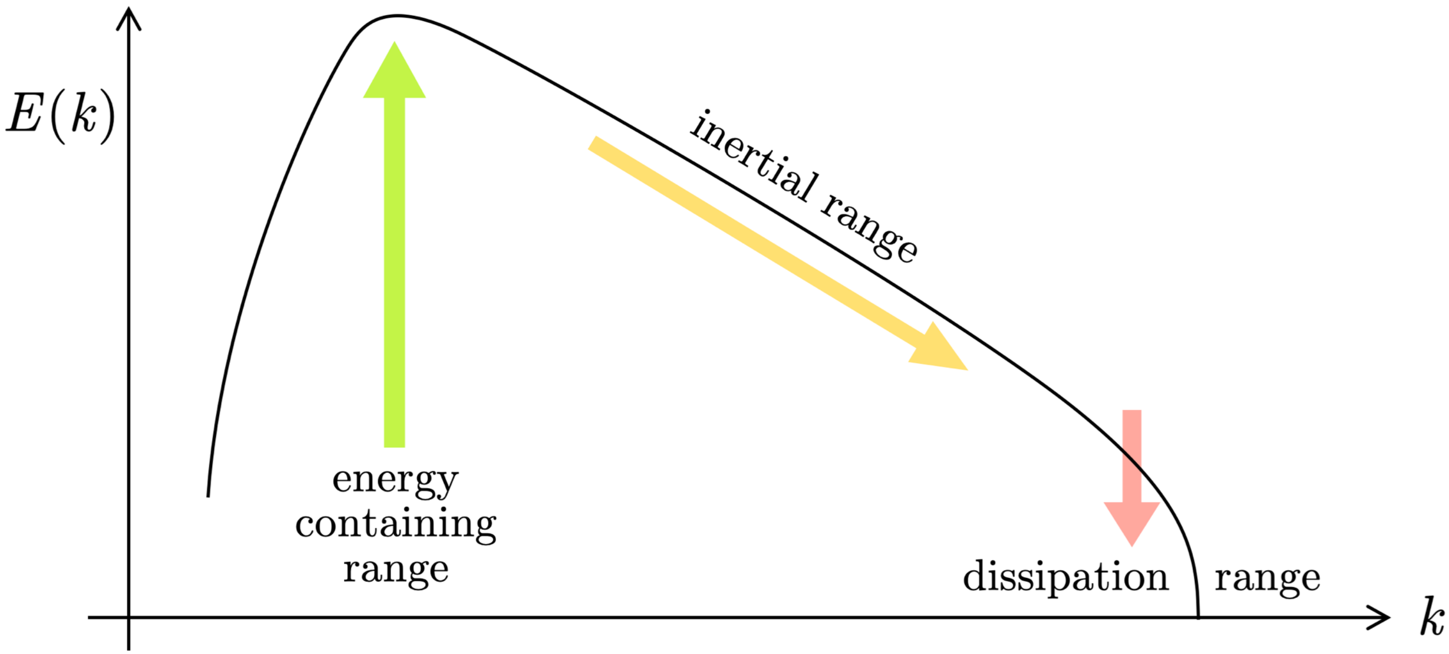
"Power" at largest scales (smallest k)

"Power" spreads to fill smaller scales (larger k)

Big whorls have little whorls
Which feed on their velocity,
And little whorls have lesser whorls
And so on to viscosity.
— *Lewis Richardson*

(6) *Turbulence*

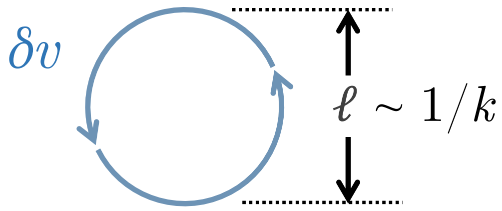
- The Fourier power spectrum reveals more physics…



- If the system is being "stirred" with a time-steady energy input, the flow of energy through the system should be time-steady, too…

$$Q_{\rm stir} = \left\{ Q_{\rm cascade} = \frac{\Delta U_{\rm eddy}}{\Delta t} \right\} = Q_{\rm damp} = Q_{\rm heat}$$

(6) *Turbulence*



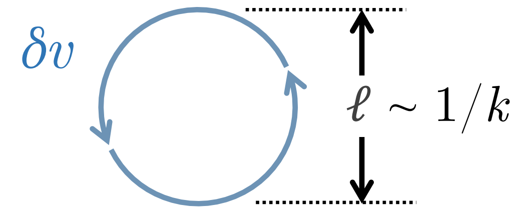
- For eddies at a given size-scale, we can assume a **1st postulate of strong turbulence:**

$$\left\{ \begin{array}{c} \text{eddy} \\ \text{circulation time} \end{array} \right\} \approx \left\{ \begin{array}{c} \text{eddy} \\ \text{lifetime} \end{array} \right\} \approx \frac{\ell}{\delta v} = \tau_{\text{nl}}$$

- This sets us apart from sinusoidal waves, for which lifetime >> 1 period.

(6) *Turbulence*

- For eddies at a given size-scale, we can assume a **1st postulate of strong turbulence:**



$$\left\{ \begin{array}{c} \text{eddy} \\ \text{circulation time} \end{array} \right\} \approx \left\{ \begin{array}{c} \text{eddy} \\ \text{lifetime} \end{array} \right\} \approx \frac{\ell}{\delta v} = \tau_{\rm nl}$$

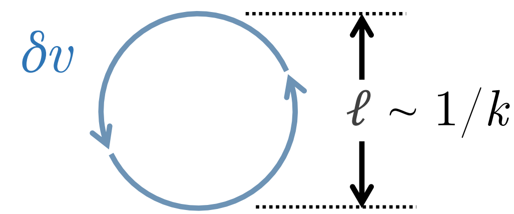
- This sets us apart from sinusoidal waves, for which lifetime >> 1 period.
- The cascade rate (which is also the heating rate, if all is time-steady) is:

$$Q_{\rm cascade} = \frac{\Delta U_{\rm eddy}}{\Delta t} = \frac{\rho(\delta v)^2}{\tau_{\rm nl}} = \frac{\rho(\delta v)^3}{\ell}$$

- The power-law shape of the inertial range ($E \propto k^{-5/3}$) comes from assuming the **2nd postulate of strong turbulence:** that $Q_{\rm cascade}$ = constant as a function of k.

(6) *Turbulence*

- For eddies at a given size-scale, we can assume a **1st postulate of strong turbulence:**



$$\left\{ \begin{array}{c} \text{eddy} \\ \text{circulation time} \end{array} \right\} \approx \left\{ \begin{array}{c} \text{eddy} \\ \text{lifetime} \end{array} \right\} \approx \frac{\ell}{\delta v} = \tau_{\rm nl}$$

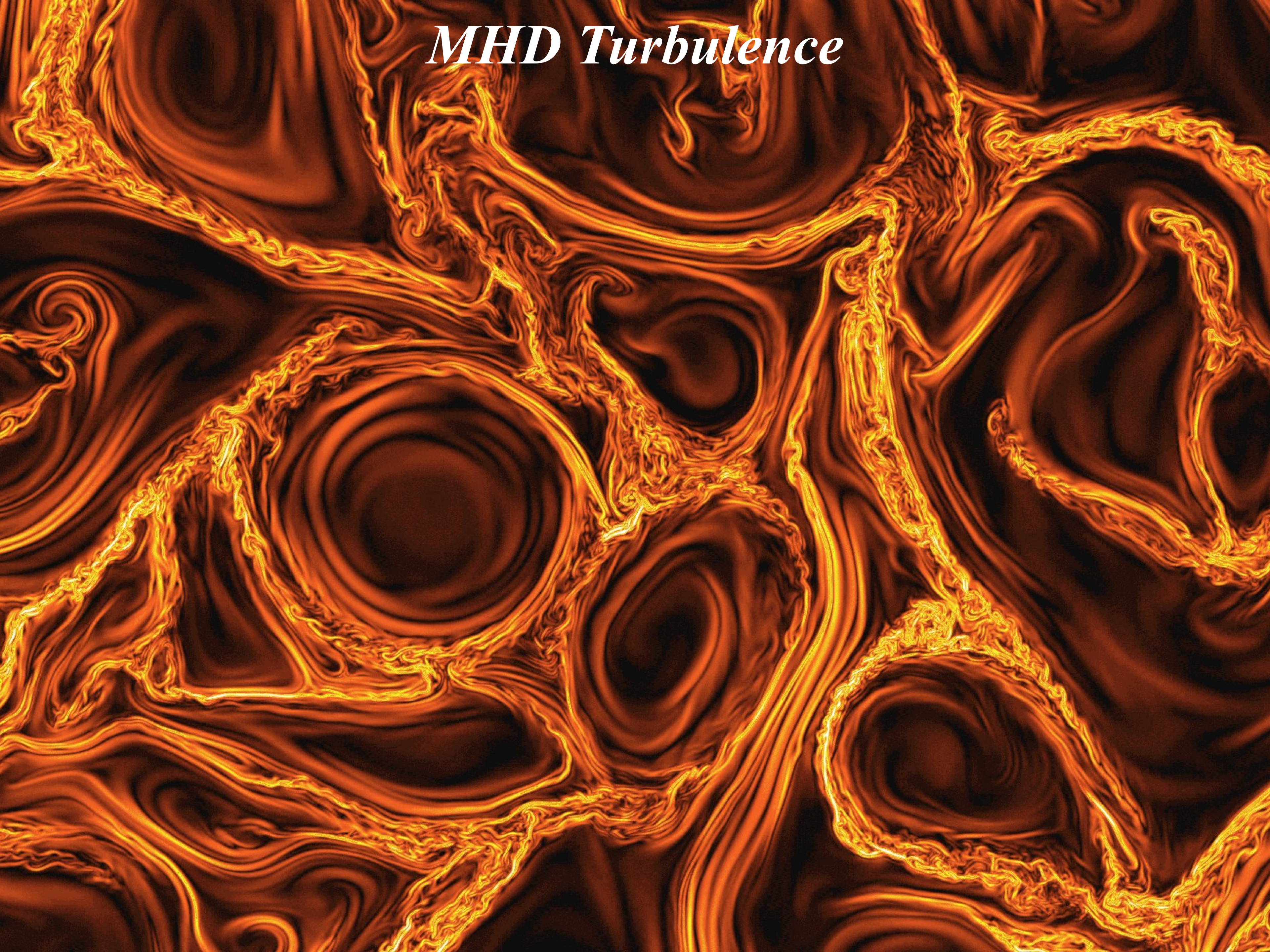
- This sets us apart from sinusoidal waves, for which lifetime >> 1 period.
- The cascade rate (which is also the heating rate, if all is time-steady) is:

$$Q_{\rm cascade} = \frac{\Delta U_{\rm eddy}}{\Delta t} = \frac{\rho(\delta v)^2}{\tau_{\rm nl}} = \frac{\rho(\delta v)^3}{\ell}$$

- The power-law shape of the inertial range ($E \propto k^{-5/3}$) comes from assuming the **2nd postulate of strong turbulence:** that $Q_{\rm cascade}$ = constant as a function of k.
- In week 2's notation:

$$\left(\begin{array}{c} \delta v = u \\ \ell = \lambda_{\rm ph} \end{array} \right) \qquad Q_{\rm heat} = \mathcal{E} \frac{\rho u V_{\rm A}^2}{L} \sim \frac{\rho u^3}{\lambda_{\rm ph}} \qquad \Longrightarrow \qquad \mathcal{E} \sim \frac{L\, u^2}{\lambda_{\rm ph} V_{\rm A}^2}$$

$$m = 2$$
$$n = 1$$


MHD Turbulence

For next week

- Continue tinkering with the hands-on computation exercise; due next week (Thursday, February 10, 2022).

- Participate in the #hands-on-1-discussion channel on Slack, even if just to vent about python…