# ASTR-6000 Seminar
# COLLAGE: Coronal Heating, Solar Wind, & Space Weather

January 27, 2022

## Coronal heating: discussion & hands-on exercises

Dr. Steven R. Cranmer
Dr. Thomas E. Berger

# *Outline*

1. Finish discussing slides from last week ✓
2. The RTV model & Martens (2010)
3. `handson_corona_v1.ipynb`

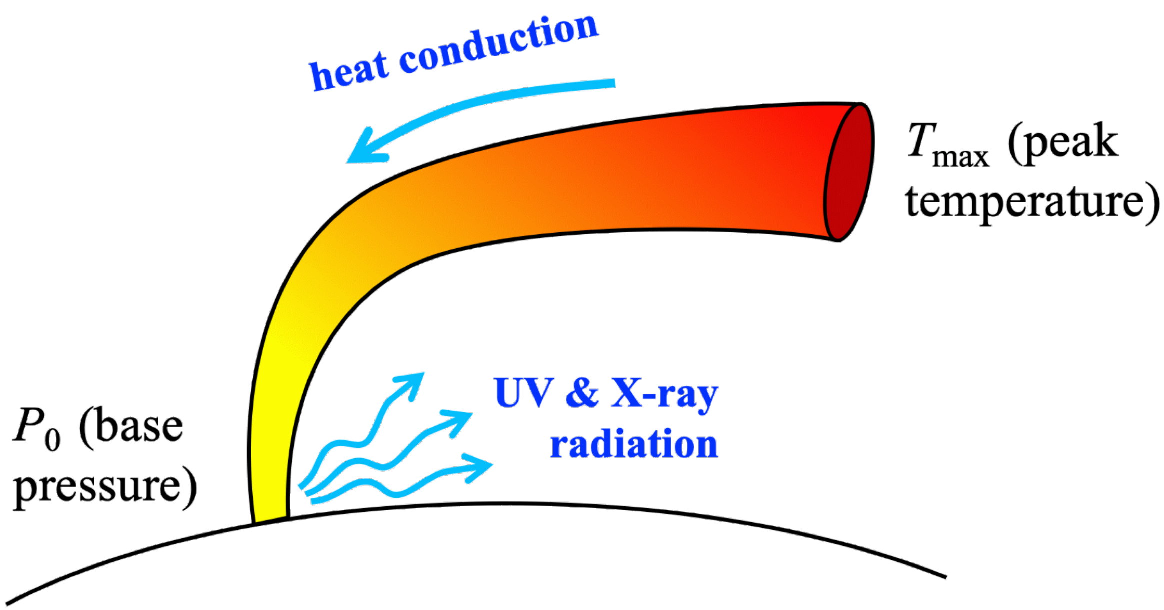
# *Coronal energy balance*

- Martens (2010) presented a mathematical way to solve the full balance between the three dominant heating/cooling terms in a static coronal loop:

$$Q_{\mathrm{cond}} + Q_{\mathrm{heat}} + Q_{\mathrm{rad}} = 0$$

- I did not assign you to read the classic "RTV" (Rosner, Tucker, & Vaiana 1978) paper, because they didn't really **show their work.**

- Here, let's show the work with a simpler version (constant heating rate: $\alpha = \beta = 0$).



# *Coronal energy balance*

- Martens (2010) assumed the standard Spitzer-Härm form for heat conduction and a power-law form for the radiative cooling function…

$$Q_{\rm cond} + Q_{\rm heat} + Q_{\rm rad} = 0$$

$$\frac{d}{dz}\left(\kappa_0 T^{5/2}\frac{dT}{dz}\right) + E_h - P_0^2 \chi_0 T^{-(2+\gamma)} = 0. \qquad (1)$$

- Let's assume $P_0$ = constant (which Martens does) and $E_h$ = constant (which he doesn't).

# *Coronal energy balance*

- Martens (2010) assumed the standard Spitzer-Härm form for heat conduction and a power-law form for the radiative cooling function…

$$Q_{\rm cond} + Q_{\rm heat} + Q_{\rm rad} = 0$$

$$\frac{d}{dz}\left(\kappa_0 T^{5/2}\frac{dT}{dz}\right) + E_h - P_0^2\chi_0 T^{-(2+\gamma)} = 0. \qquad (1)$$

- Let's assume $P_0$ = constant (which Martens does) and $E_h$ = constant (which he doesn't).
- Define dimensionless variables…

$$\eta = \left(\frac{T}{T_{\rm max}}\right)^{7/2}, \quad x = \frac{z}{L}, \quad \epsilon = \frac{2\kappa_0 T_{\rm max}^{(11/2)+\gamma}}{7\chi_0 P_0^2 L^2}, \quad \xi = \frac{E_h T_{\rm max}^{2+\gamma}}{\chi_0 P_0^2}, \quad \mu = -\frac{2(2+\gamma)}{7}$$

$$\epsilon\frac{\partial^2\eta}{\partial x^2} + \xi - \eta^\mu = 0$$

(and we see why adding $Q_{\rm rad}$ makes it a more difficult equation to solve)

# *Coronal energy balance*

$$\epsilon \frac{\partial^2 \eta}{\partial x^2} + \xi - \eta^\mu = 0$$

- When we neglected radiation, we had a 2nd order differential equation with 3 boundary conditions. The "extra" one helped us pin down the constant ($\xi / \epsilon$).

# *Coronal energy balance*

$$\epsilon \frac{\partial^2 \eta}{\partial x^2} + \xi - \eta^{\mu} = 0$$

- When we neglected radiation, we had a 2nd order differential equation with 3 boundary conditions. The "extra" one helped us pin down the constant ($\xi / \epsilon$).
- Here, we want to solve for both $\xi$ and $\epsilon$. Thus, we need a **4th** boundary condition…

1. $\eta(1) = 1$ (at the top of the loop, we've defined $T$ to be at its maximum value)
2. $\eta'(1) = 0$ (there should be symmetry at the top, with $T$ remaining smooth)
3. $\eta(0) \approx 0$ (note that $(10^4/10^6)^{7/2} \approx 10^{-7}$, which is pretty close to zero)
4. $\eta'(0) \approx 0$ (essentially assumes that $\mathbf{q}_{\mathrm{cond}}$ is negligible at the base, too) ☜

# *Coronal energy balance*

$$\epsilon \frac{\partial^2 \eta}{\partial x^2} + \xi - \eta^\mu = 0$$

- When we neglected radiation, we had a 2nd order differential equation with 3 boundary conditions. The "extra" one helped us pin down the constant ($\xi / \epsilon$).
- Here, we want to solve for both $\xi$ and $\epsilon$. Thus, we need a **4th** boundary condition…

1. $\eta(1) = 1$ (at the top of the loop, we've defined $T$ to be at its maximum value)
2. $\eta'(1) = 0$ (there should be symmetry at the top, with $T$ remaining smooth)
3. $\eta(0) \approx 0$ (note that $(10^4/10^6)^{7/2} \approx 10^{-7}$, which is pretty close to zero)
4. $\eta'(0) \approx 0$ (essentially assumes that $\mathbf{q}_{cond}$ is negligible at the base, too) ☜

- Define $q = d\eta/dx = \eta'$ then multiply both sides by $q$. Integrate over $x$, and use tricks…

$$\int dx\; q\, \frac{dq}{dx} = \int q\, dq = \frac{q^2}{2} \qquad \text{and} \qquad \int dx\; q\; f(\eta) = \int d\eta\; f(\eta)$$

# *Coronal energy balance*

$$\epsilon \frac{\partial^2 \eta}{\partial x^2} + \xi - \eta^{\mu} = 0$$

$$\int dx\; q\, \frac{dq}{dx} = \int q\, dq = \frac{q^2}{2} \qquad \text{and} \qquad \int dx\; q\; f(\eta) = \int d\eta\; f(\eta)$$

- If we do these integrals and use both bottom boundary conditions ($\eta(0) = 0$, $\eta'(0) = 0$), we get

$$\frac{\epsilon\, q^2}{2} = \frac{\eta^{\mu+1}}{\mu+1} - \xi\eta$$

# *Coronal energy balance*

$$\epsilon \frac{\partial^2 \eta}{\partial x^2} + \xi - \eta^\mu = 0$$

$$\int dx\, q\, \frac{dq}{dx} = \int q\, dq = \frac{q^2}{2} \qquad \text{and} \qquad \int dx\, q\, f(\eta) = \int d\eta\, f(\eta)$$

- If we do these integrals and use both bottom boundary conditions ($\eta(0) = 0$, $\eta'(0) = 0$), we get

$$\frac{\epsilon\, q^2}{2} = \frac{\eta^{\mu+1}}{\mu+1} - \xi\eta$$

- We can integrate one more time to get $\eta$ as a function of $x$, but note that if we apply the top boundary conditions ($\eta(1) = 1$, $\eta'(1) = q(1) = 0$), to this, we get

$$\xi = \frac{1}{\mu+1}$$

which gives us one of our main unknowns (i.e., one of the two "RTV scaling laws")

# *Coronal energy balance*

- How to integrate one more time? Note that

$$\frac{\epsilon\, q^2}{2} = \frac{\eta^{\mu+1}}{\mu+1} - \xi\eta$$

is equivalent to

$$q = \frac{d\eta}{dx} = \sqrt{\frac{2}{\epsilon}\left(\frac{\eta^{\mu+1}}{\mu+1} - \frac{\eta}{\mu+1}\right)}$$

# *Coronal energy balance*

- How to integrate one more time? Note that

$$\frac{\epsilon\, q^2}{2} = \frac{\eta^{\mu+1}}{\mu+1} - \xi\eta$$

is equivalent to

$$q = \frac{d\eta}{dx} = \sqrt{\frac{2}{\epsilon}\left(\frac{\eta^{\mu+1}}{\mu+1} - \frac{\eta}{\mu+1}\right)}$$

and we can integrate over the whole loop ($x$ = 0 to 1) to get

$$\int_0^1 \frac{d\eta}{\sqrt{\eta^{\mu+1} - \eta}} = \sqrt{\frac{2}{\epsilon(\mu+1)}} \int_0^1 dx$$

# *Coronal energy balance*

- How to integrate one more time? Note that

$$\frac{\epsilon\, q^2}{2} = \frac{\eta^{\mu+1}}{\mu+1} - \xi\eta$$

is equivalent to

$$q = \frac{d\eta}{dx} = \sqrt{\frac{2}{\epsilon}\left(\frac{\eta^{\mu+1}}{\mu+1} - \frac{\eta}{\mu+1}\right)}$$

and we can integrate over the whole loop ($x$ = 0 to 1) to get

$$\underbrace{\int_0^1 \frac{d\eta}{\sqrt{\eta^{\mu+1} - \eta}}}_{\text{Just a number!}} = \sqrt{\frac{2}{\epsilon(\mu+1)}}\ \underbrace{\int_0^1 dx}_{\text{Just 1 !}}$$

Just a number!

Just 1 !

For $\gamma = 0.5$, $\mu = -5/7$, and the integral is ≈ 2.51199…

This gives us a value for $\epsilon$. For $\gamma = 0.5$, $\epsilon \approx 1.1093\ldots$

# *Coronal energy balance*

- Having values for both $\epsilon$ and $\xi$ allows us to specify the two RTV scaling laws.
- One sees them in different forms. Coming directly from the constants, we get

$$P_0^2\, L^2 \;\propto\; T_{\rm max}^{(11/2)+\gamma} \qquad \text{and} \qquad P_0^2 \;\propto\; E_h\, T_{\rm max}^{2+\gamma}$$

# *Coronal energy balance*

- Having values for both $\epsilon$ and $\xi$ allows us to specify the <u>two</u> RTV scaling laws.
- One sees them in different forms. Coming directly from the constants, we get

$$P_0^2\, L^2 \;\propto\; T_{\rm max}^{(11/2)+\gamma} \qquad \text{and} \qquad P_0^2 \;\propto\; E_h\, T_{\rm max}^{2+\gamma}$$

- However, I prefer to rearrange… to solve for things we **don't know** in terms of things we **do know…**

$$T_{\rm max} \;\propto\; E_h^{2/7}\, L^{4/7} \qquad \text{and} \qquad P_0 \;\propto\; E_h^{(11+2\gamma)/14}\, L^{(4+2\gamma)/7}$$

SAME scaling relation that we got from assuming

$$Q_{\rm heat} + Q_{\rm cond} = 0$$

# *Coronal energy balance*

- Martens (2010) adds in the wrinkle of a heating rate that depends on position... indirectly, via $T$ and $P$ (or $T$ and $\rho$). (These are the α and β exponents)
- There may also be flux-tube expansion from base to apex… this is included as the δ exponent.
- Martens also shows how the position dependence of all quantities can be found by integrating ***not*** all the way from 0 to 1.
- In the hands-on python notebook, I also show how all this can be written in terms of my $Q_{\rm heat}$ (i.e., Poynting flux times an efficiency factor):

$$H P^b T^a = H \left( \frac{k_{\rm B}}{\mu m_{\rm H}} \right)^b \rho^b T^{a+b} = \mathcal{E}_0 \, \lambda_{\rm ph}^{n-m} L^{m-n-1} u_{\rm ph}^{m+1} \left( B_0/\sqrt{4\pi} \right)^{2-m} \rho^{m/2} \left( \frac{T_0}{T} \right)^{\delta(2-b)}$$

Too many symbols! This isn't Martens' $\mu$. It's the mean mass per particle, in units of hydrogen. For the fully ionized corona, $\mu \approx 0.6$.

# *The python notebook*

`handson_corona_v1.ipynb`

- Pinned at the top of the #paper-2-discussion channel on the Slack.
- Also posted on course web page (under today's date in schedule).

# *For next week*

- Work on your **extension** of what's in the notebook… e.g., answering questions posed therein, making modifications, adding new physics, re-implementing in a better programming language, etc… really whatever you would like to do that helps you understand coronal heating better.
- Due in 2 weeks (Thursday, February 10, 2022)
- I'll create a Slack channel for discussions about this work, but if you want to just submit your final result to me via email (or Canvas for CU students), that's fine, too.