ASTR-6000 Seminar COLLAGE: Coronal Heating, Solar Wind, & Space Weather

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Coronal heating: origins & scaling laws

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Outline

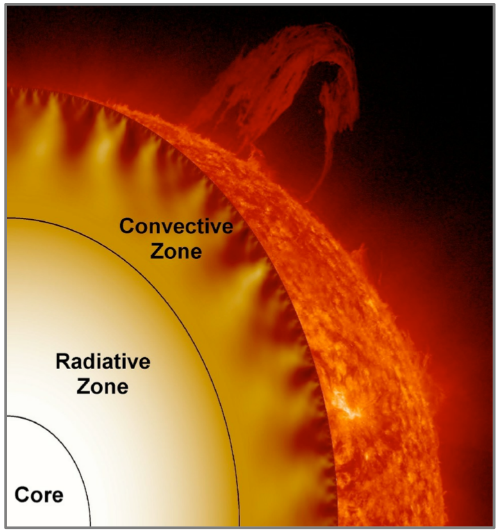
1. **Physical Origins**
    a. How much energy is available?
    b. How much energy is transported up (Poynting flux)?
    c. A plethora of processes proposed to prevail over the problem

2. **How does heating determine the ~steady-state ρ, T, P ?**
    a. Thermal energy conservation: heating balances cooling
    b. Photosphere (radiation alone)
    c. Chromosphere (radiation + heating)
    d. Inner corona (conduction + radiation + heating)
    e. Outer corona (conduction alone?)

(1a) *How much energy is available?*

- The ultimate energy source for the chromosphere, corona, and solar wind is generally understood to be the kinetic energy of convective motions.

- Inside the convection zone, convective eddies carry **nearly all** of the Sun's energy flux:



$$F_\odot = \frac{L_\odot}{4\pi R_\odot^2} = \sigma T_{\rm eff}^4 \approx 63{,}000 \text{ kW/m}^2$$

- However, the Sun becomes convectively **stable** just below the photosphere, so the eddies become weaker as they "overshoot" the convection zone by a bit.

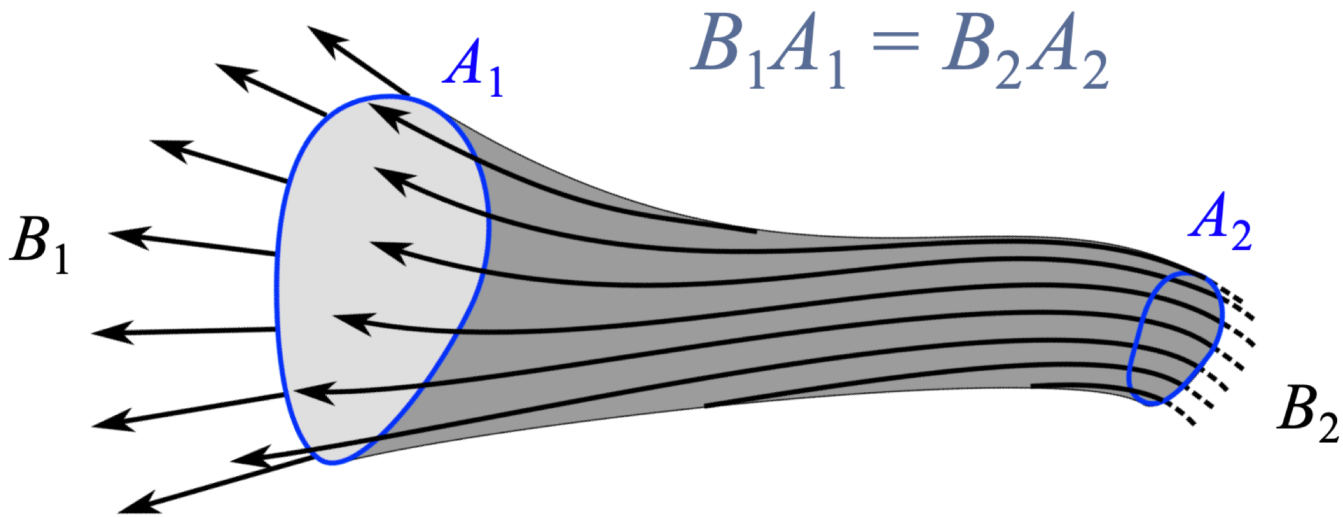
- We can estimate the photospheric kinetic energy flux from typical observed densities (a few × 10^{-7} g/cm^3) and granulation flow speeds (1–2 km/s)…

$$F_{\rm phot} \approx \frac{1}{2}\rho u^3 \approx 500 \text{ kW/m}^2$$

- This doesn't distinguish between actual up/down flows (that deliver thermal energy from below) and horizontal motions (that maintain mass conservation).

(1a) *How much energy is available?*

- Also, it seems clear that only a small **filling factor** (*f*) of the photospheric surface is connected magnetically to the corona.
- Energy flux (power per unit area) gets diluted if the area expands…
- A key consequence of Maxwell's most boring equation ($\nabla \cdot \mathbf{B} = 0$) is that **magnetic flux is conserved** for a bundle of field lines…



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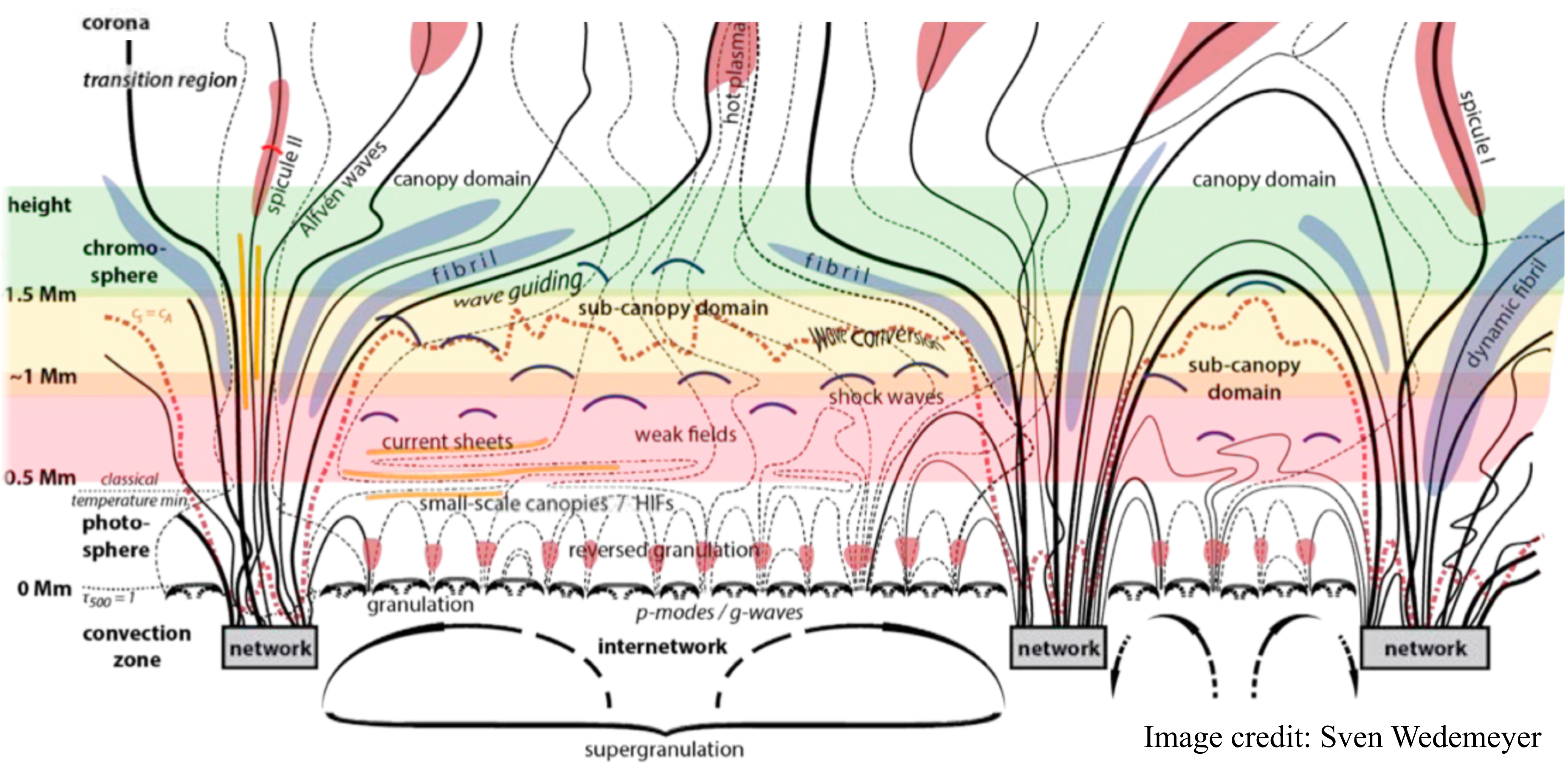

Image credit: Sven Wedemeyer

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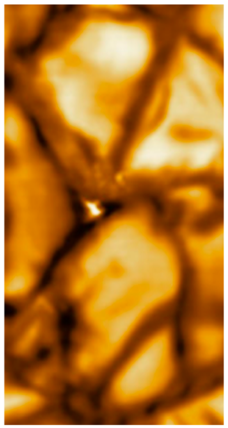
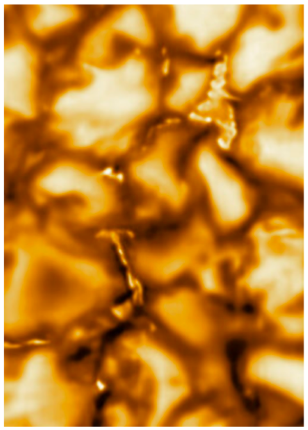
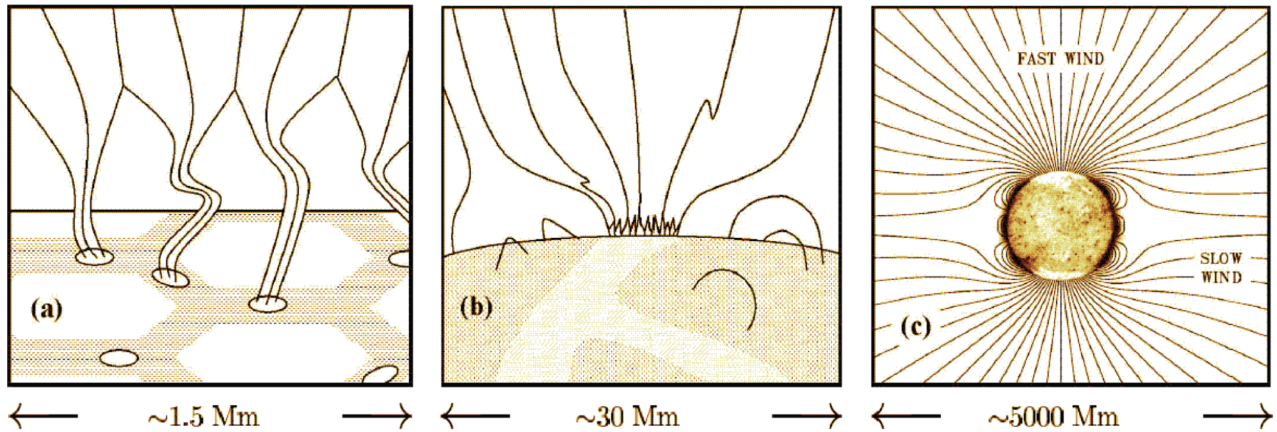






- B in intergranular bright points: 1000 to 2000 G.
- mean B in ARs: a few 100 to *maybe* 1000 G. $\rightarrow$ $f \approx 0.1$
- mean B in QS & CH: a few to tens of G. $\rightarrow$ $f \approx 0.01$

$F_{AR} \approx 50$ kW/m^2

$F_{QS} \approx 5$ kW/m^2

(1a) *How much energy is available?*

- Available: $F_{AR} \approx 5$ to 50 kW/m^2
- What do we *need* in order to heat the chromosphere and corona?
  We'll figure it out ourselves, but for a preview, see Withbroe & Noyes (1977):

Table 1 Chromospheric and coronal energy losses

Parameter	Quiet Sun	Coronal hole	Active region
Transition layer pressure (dyn cm^{-2})	2×10^{-1}	7×10^{-2}	2
Coronal temperature (K, at $r \approx 1.1\ R_\odot$)	1.1 to 1.6×10^6	10^6	2.5×10^6
Coronal energy losses (kW m^{-2})			
Conduction flux F_c	0.2	0.06	0.1 to 10
Radiative flux F_r	0.1	0.01	5
Solar wind flux F_w	< 0.05	0.7	(< 0.1)
Total corona loss $F_c + F_r + F_w$	0.3	0.8	10
Chromospheric radiative losses (kW m^{-2})			
Low chromosphere	2	2	> 10
Middle chromosphere	2	2	10
Upper chromosphere	0.3	0.3	2
Total chromospheric loss	4	4	20
Solar wind mass loss (g cm^{-2} sec^{-1})	$\lesssim 2 \times 10^{-11}$	2×10^{-10}	$(< 4 \times 10^{-11})$

(1b) *How much energy is transported up?*

- If we were dealing with energy flow purely in **E&M fields,** Maxwell's equations can be converted into an energy conservation equation:

$$\frac{\partial}{\partial t}(U_{\mathrm{E}} + U_{\mathrm{B}}) + \nabla \cdot \mathbf{S} \;=\; -\mathbf{J} \cdot \mathbf{E}$$

$$U_{\mathrm{E}} = \frac{|\mathbf{E}|^2}{8\pi} \;, \qquad U_{\mathrm{B}} = \frac{|\mathbf{B}|^2}{8\pi} \qquad \text{(electric \& magnetic energy densities)}$$

$$\mathbf{S} = \frac{c}{4\pi}(\mathbf{E} \times \mathbf{B}) \qquad \text{(Poynting flux)} \;\; .$$

i.e., a parcel can undergo time evolution of its E&M energy only when

a. there's a transport of energy flux into or out of it, or

b. there's local dissipation due to electric currents ("Joule heating")

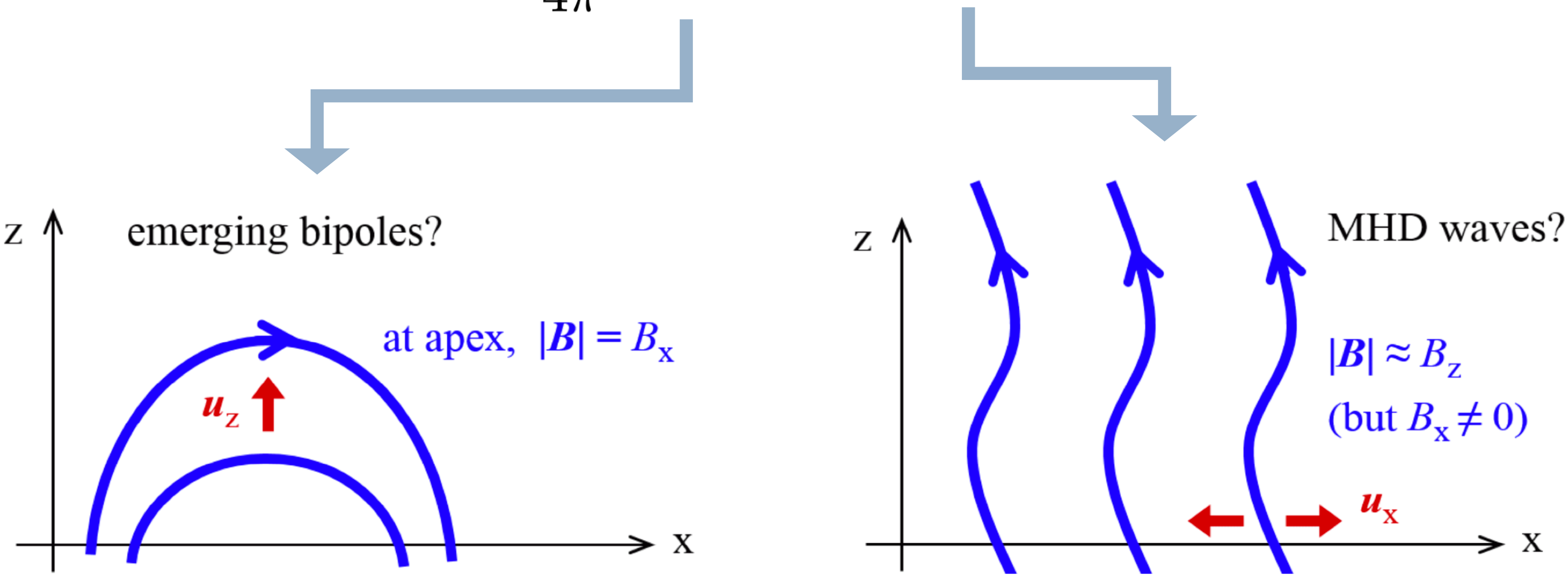
- For an **MHD magnetofluid** (i.e., high-conductivity plasma embedded in magnetic field), the Poynting flux still tells us a lot about how energy carried by **B** is transported…

$$\mathbf{E} \;=\; -\frac{\mathbf{u}}{c} \times \mathbf{B} \qquad \text{so, the Poynting flux is} \qquad \mathbf{S} \;=\; \frac{1}{4\pi}\left[\mathbf{B} \times (\mathbf{u} \times \mathbf{B})\right] \;\; .$$

(1b) *How much energy is transported up?*

- Examining a small piece of the Sun so the surface is ~a flat plane, we can evaluate the *vertical* component of the Poynting flux [components: vertical (z) & horizontal (x)]:

$$S_z = \frac{1}{4\pi}\left[u_z B_x^2 - (\mathbf{u}_x \cdot \mathbf{B}_x) B_z\right]$$



- On the Sun, is the Poynting flux dominated by the 1st term (flux emergence), or by the 2nd term (oblique fields jostled horizontally)? Unclear! **Both** probably contribute.

(1b) *How much energy is transported up?*

- Putting aside vector directions, does the Poynting flux give us the right order of magnitude for the energy flux we know is present?

$$S \sim \frac{u\,B^2}{4\pi} \sim \rho\, u\, V_A^2 \qquad \text{since} \qquad V_A = \frac{B}{\sqrt{4\pi\rho}}$$

For u = 1 km/s and B = (10, 30, 100) G, we get ***S*** **≈ (0.8, 8, 80) kW/m²**.

- It's okay that it's *a bit* larger than needed.
- Strictly speaking, S is the energy flux flowing "in" the magnetic field.
- A big part of the coronal heating problem is figuring out how a fraction of the available magnetic energy gets **transferred to the particles** (i.e., converted into thermal, or random kinetic, energy).
- Thus, let's put much of what we don't know into a dimensionless efficiency factor, and define the heating rate ~ $\partial U/\partial t$ (in units of power per unit volume) as…

$$Q_{\text{heat}} = \mathcal{E}\,|\nabla \cdot \mathbf{S}| \sim \mathcal{E}\,\frac{u\,B^2}{4\pi\,L}$$

(1b) *How much energy is transported up?*

$$Q_{\rm heat} \;=\; \mathcal{E}\,|\nabla\cdot\mathbf{S}| \;\sim\; \mathcal{E}\,\frac{u\,B^2}{4\pi\,L}$$

When writing a recent review paper (Cranmer & Winebarger 2019), we realized that the physics in many proposed **coronal heating processes** could be written as…

$$\mathcal{E} \;\approx\; \mathcal{E}_0 \left(\frac{\lambda_{\rm ph}}{L}\right)^n \left(\frac{\tau_{\rm A}}{\tau_{\rm ph}}\right)^m$$

where

- L = length of the coronal loop
- $\lambda_{\rm ph}$ = typical horizontal scale of photospheric driving motions
- $\tau_{\rm A} = (L\,/\,V_{\rm A})$ = time scale for Alfvén waves to propagate along the loop
- $\tau_{\rm ph} = (\lambda_{\rm ph}\,/\,u)$ = typical time scale of photospheric driving motions
- n, m are given specifically by each proposed physical process

Thus,

$$Q_{\rm heat} \;\approx\; \mathcal{E}_0\,\lambda_{\rm ph}^{n-m}\,L^{m-n-1}\,u^{m+1}\,\left(B/\sqrt{4\pi}\right)^{2-m}\,\rho^{m/2}$$

(1b) *How much energy is transported up?*

$$Q_{\rm heat} \approx \mathcal{E}_0 \, \lambda_{\rm ph}^{n-m} \, L^{m-n-1} \, u^{m+1} \, \left(B/\sqrt{4\pi}\right)^{2-m} \rho^{m/2}$$

To think about: if increasing either *u* or *B* makes for more heating, then what is a realistic range of values for the exponent *m* ?

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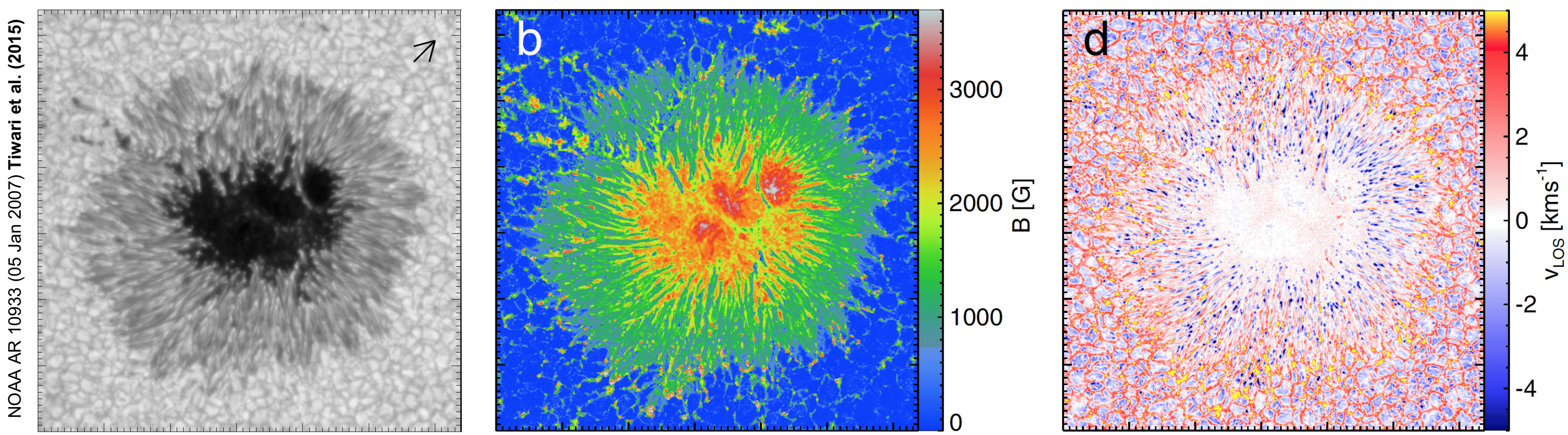
To think about: if increasing either *u* or *B* makes for more heating, then what is a realistic range of values for the exponent *m* ?

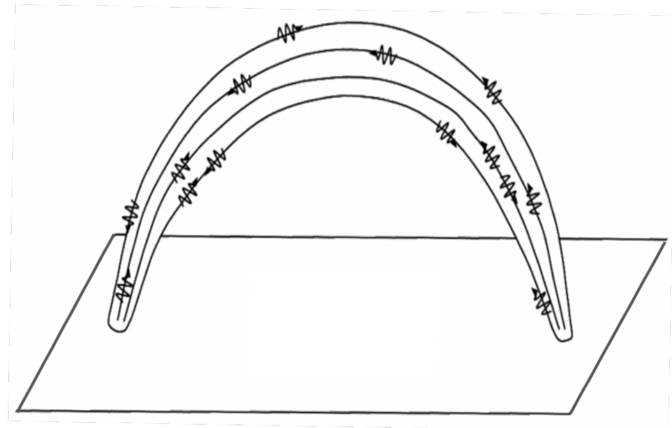
$$\left.\begin{array}{lll} m+1 > 0 & \rightarrow & m > -1 \\ 2-m > 0 & \rightarrow & m < +2 \end{array}\right\} \rightarrow \quad -1 < m < +2$$

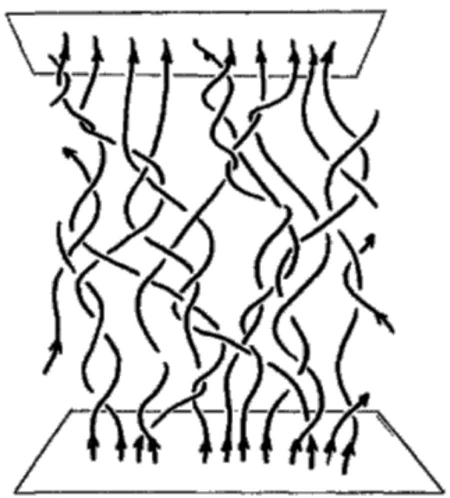
However… what if *u* depends on *B* ?



(1c) *Proposed coronal heating processes*

- If the driving motions are **"fast"** (i.e., $\tau_{ph} \ll \tau_A$), then those motions are able to propagate along the field in the form of waves.
- Heating occurs when those waves are damped out by particle collisions (e.g., viscosity, resistivity).
- These processes resemble **AC ("alternating current")** systems.

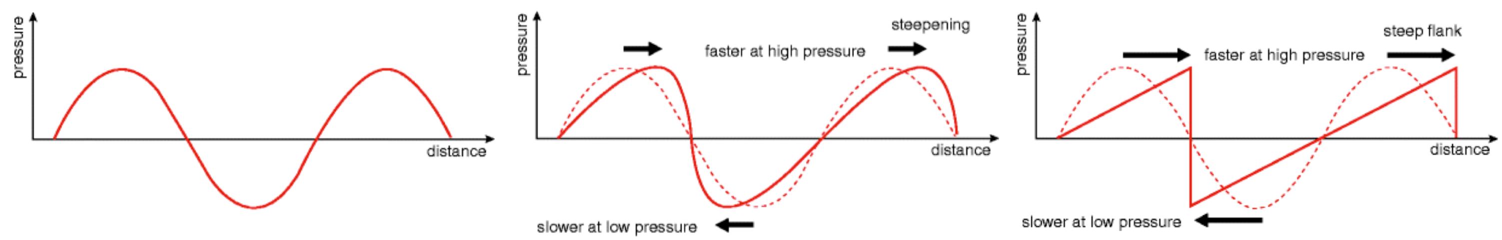




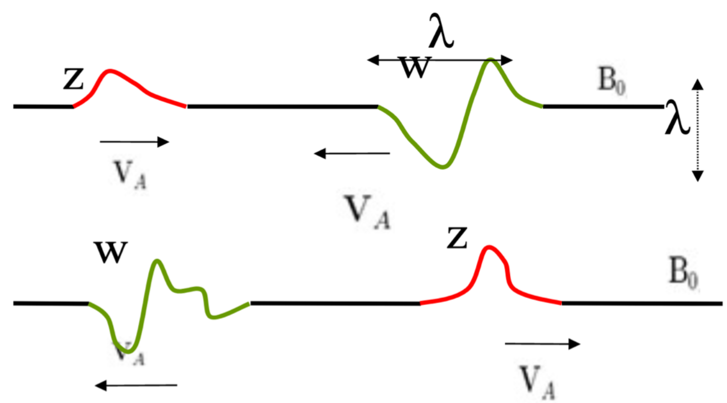
- If the driving motions are **"slow"** (i.e., $\tau_{ph} \gg \tau_A$), then the B-field has plenty of time to relax in response. The field gets slowly tangled & braided.
- Heating occurs when those tangles build up to little stress-points of instability, and magnetic reconnection is triggered making lots of little "nanoflares."
- These processes resemble **DC ("direct current")** systems.

(1c) *Proposed coronal heating processes*

- Many parts of the corona have $\tau_{ph} \sim \tau_A$! It's neither cleanly AC nor DC.
- When things get messy, fluctuations in real MHD plasmas tend to become nonlinear…
  - Acoustic-like **waves** "steepen" into **shocks:**



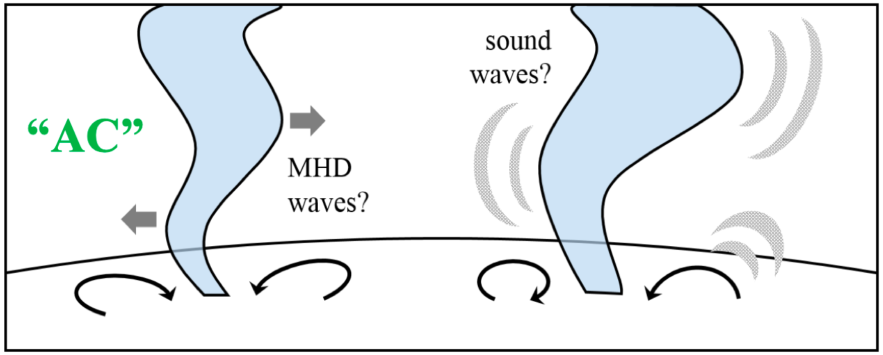
  - Alfven-like **wave** packets crash into one another & generate a **turbulent cascade:**
  - Or, **waves** interact with slowly-varying background conditions & transform via **mode coupling.**

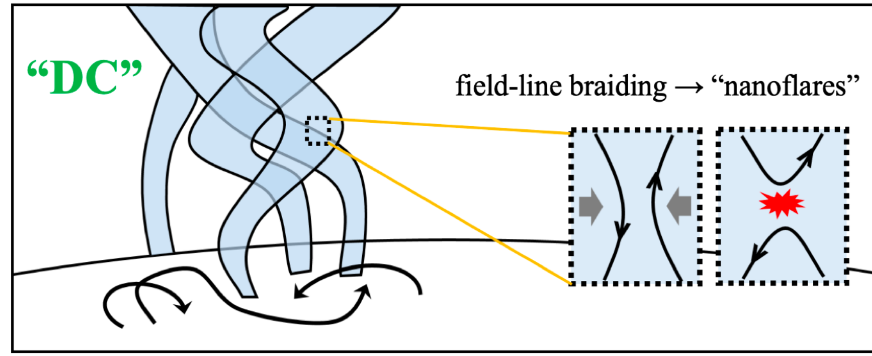


- In all cases, **smaller scales** get spontaneously generated from larger scales… they're much easier to "damp" to heat the plasma! If intermittent, heating is still **"nanoflarish."**

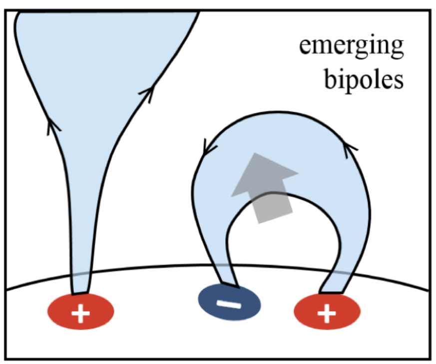
(1c) *Proposed coronal heating processes*

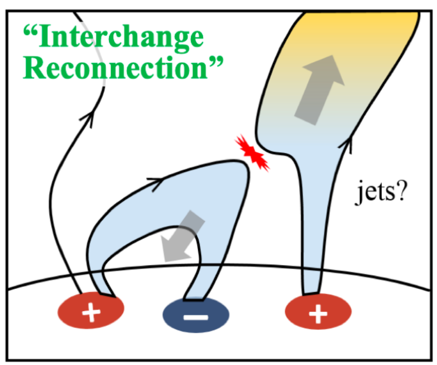
Graphical summary…

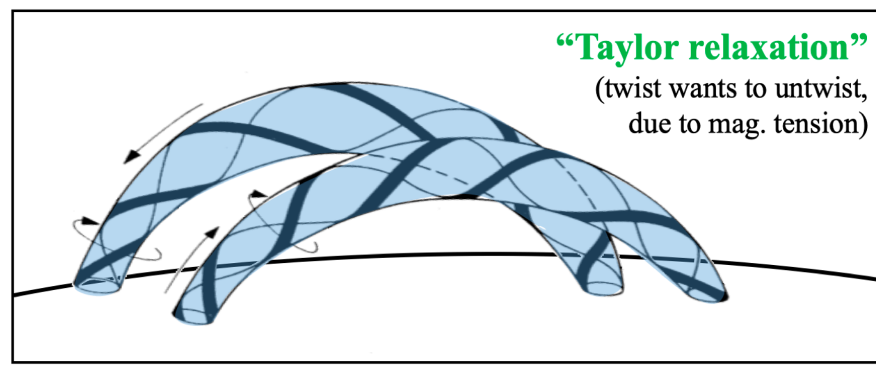




Plus two other processes that we’ll discuss a bit later…







Probably not applicable to the ***entire*** corona…

Only important in twisted **(“sigmoid”)** active regions?

(1c) *Proposed coronal heating processes*

$Q \approx \mathcal{E}\, \rho V_A^2 v_\perp / L$ $\Lambda = \lambda_\perp / L$, $\Theta = \tau_A / \tau_{ph}$

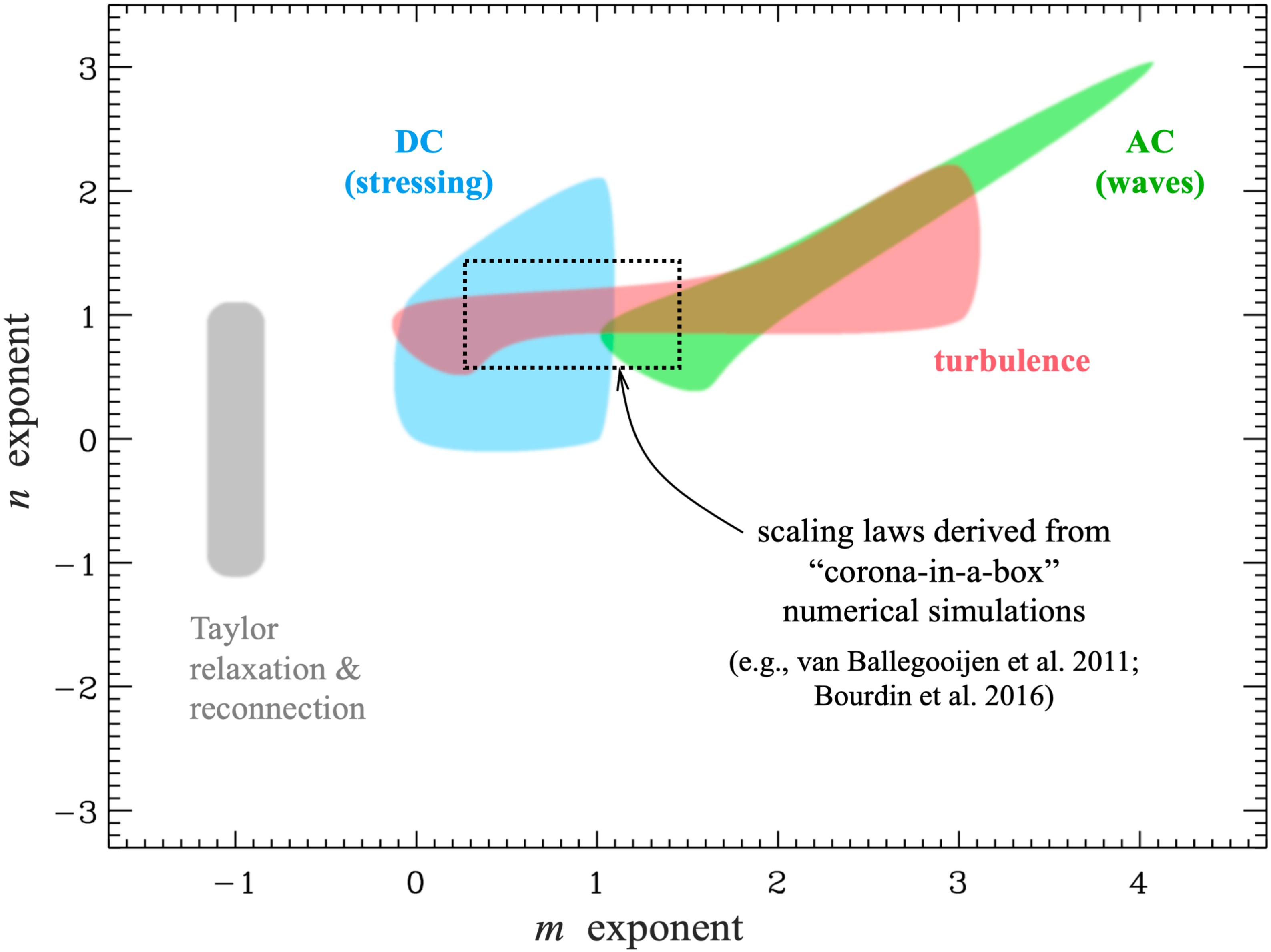
AC: alternating current, $\tau_A \gg \tau_{ph}$

DC: direct current, $\tau_A \ll \tau_{ph}$

Model description	**Efficiency ($\mathcal{E}$)**	**Example reference**
	Wave Dissipation (AC) Models	
Alfvén-wave collisional damping	$\Lambda^1 \Theta^2 Re^{-1}$	Osterbrock (1961)
Resonant absorption	$\Lambda^1 \Theta^1$	Ruderman et al. (1997)
Phase mixing	$\Lambda^1 \Theta^{4/3} Re^{-1/3}$	Roberts (2000)
Surface-wave damping	$\Lambda^{1/2} \Theta^{3/2} (\Sigma / Re)^{1/2}$	Hollweg (1985)
Fast-mode shock train	$\Lambda^2 \Theta^3$	Hollweg (1985)
Switch-on MHD shock train	$\Lambda^3 \Theta^4$	Hollweg (1985)
	Turbulence Models	
Kolmogorov-Obukhov cascade	$\Lambda^1 \Theta^2$	Hollweg (1986)
Iroshnikov-Kraichnan cascade	$\Lambda^2 \Theta^3$	Chae et al. (2002)
Hybrid triple-correlation cascade	$\Lambda^1 \Theta^3 (1+\Theta)^{-1}$	Zhou & Matthaeus (1990)
Reflection-driven cascade	$\Lambda^1 \Theta^2 (f_+^2 f_- + f_-^2 f_+)$	Hossain et al. (1995)
2D boundary-driven cascade	$\Lambda^{2/3} \Theta^{1/3}$	Heyvaerts & Priest (1992)
Line-tied reduced MHD cascade	$\Lambda^1 \Theta^{1/2}$	Dmitruk & Gómez (1999)
	Footpoint Stressing (DC) Models	
Current-layer random walk	Λ^1	Sturrock & Uchida (1981)
Current-layer shearing	$\Lambda^1 (1+\Theta^2)^{1/2} (1+\Lambda^2)^{-1/2}$	Galsgaard & Nordlund (1996)
Braided discontinuities	$\Lambda^2 \Theta^1$	Parker (1983)
Flux cancellation	$\Lambda^1 \Theta^1 (\phi^{8/3} - \phi^{4/3})$	Priest et al. (2018)
	Taylor Relaxation Models	
Tearing-mode reconnection	$\Lambda^1 \Theta^1 (1-\alpha L)^{-5/2}$	Browning & Priest (1986)
Hyperdiffusive reconnection	$\Lambda^1 \Theta^{-1} (\alpha L)^2$	van Ballegooijen & Cranmer (2008)
Non-ideal/slipping reconnection	$\Theta^{-1} (\alpha L)^1$	Yang et al. (2018)

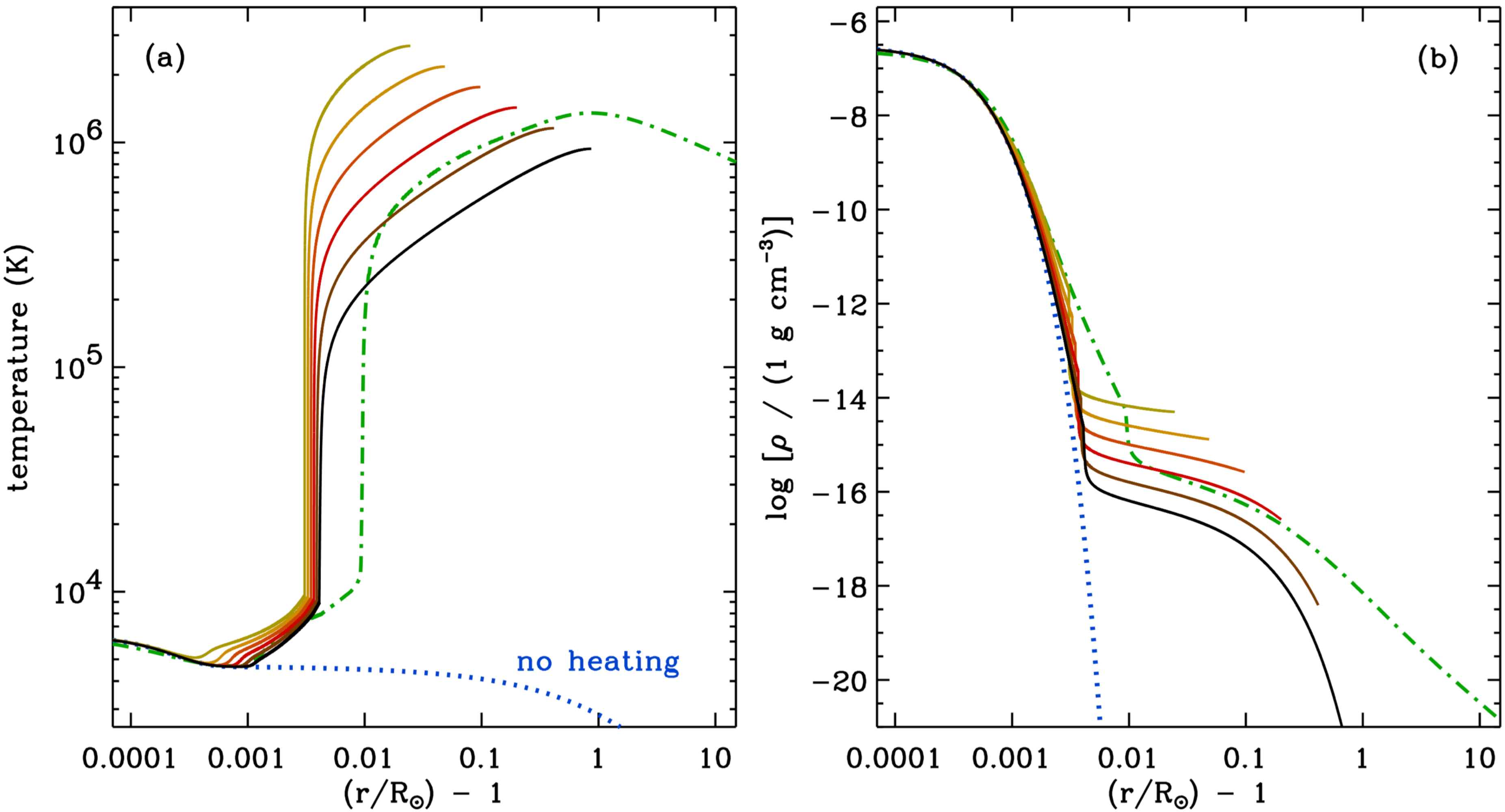
(Cranmer & Winebarger 2019, *Annual Review of Astron. & Astrophys.*, **57**, 157−187, arXiv:1811.00461)

(1c) *Proposed coronal heating processes*



(2a) *Solving energy conservation for ρ, T, P*

- Once we know how the heating happens, *how* does it affect the plasma?



(2a) *Solving energy conservation for ρ, T, P*

- For an MHD fluid, the equation of thermal energy conservation can be written as

$$\frac{\partial U_{\rm th}}{\partial t} = Q_{\rm adv} + Q_{\rm rad} + Q_{\rm cond} + Q_{\rm heat}$$

$$U_{\rm th} = \frac{3}{2}P = \frac{3}{2}n_{\rm tot}k_{\rm B}T$$

(thermal energy density; ignores ionization)

$$Q_{\rm adv} = -\frac{3u_r}{2}\frac{\partial P}{\partial r} - \frac{5P}{2A}\frac{\partial}{\partial r}(u_r A)$$

(advection & "adiabatic cooling," ignore for loops)

$$Q_{\rm rad} = \begin{cases} 4\pi\chi_{\rm eff}(J-S)\ , & \text{(optically thick)} \\ -n_e n_{\rm H}\,\Lambda(T)\ , & \text{(optically thin)} \end{cases}$$

(radiative "losses:" heating *or* cooling)

$$Q_{\rm cond} = -\nabla\cdot\mathbf{q}_{\rm cond} = \frac{1}{A}\frac{\partial}{\partial r}\left(A\,K_0\,T^{5/2}\frac{\partial T}{\partial r}\right)$$

(heat conduction; dominated by electrons)

- Many of those Q terms can be written in terms of fluxes as ($\nabla\cdot\mathbf{F}$)… but not all!
- If time-steady, $\partial U_{\rm th}/\partial t = 0$, so we solve for conditions where "heating balances cooling."

(2a) *Solving energy conservation for ρ, T, P*

$$\frac{\partial U_{\rm th}}{\partial t} = Q_{\rm adv} + Q_{\rm rad} + Q_{\rm cond} + Q_{\rm heat}$$

- It's usually never the case that all 4 terms on the right-hand side are equally important.
- The terms "in charge" shift depending on where we look…

	(2b) Photosphere	(2c) Chromosphere	Transition Region	(2d) Inner Corona	(2e) Outer Corona & Wind
$Q_{\rm rad}$	🌞	🌞	🌞		
$Q_{\rm heat}$		🌞	🌞	🌞	?
$Q_{\rm cond}$			🌞	🌞	🌞
$Q_{\rm adv}$				?	?

(2b) *Photospheric energy balance*

- Here, the term with the largest magnitude is due to radiation alone…
  but there could be *either* radiative heating *or* radiative cooling.

- It depends on which atomic processes (emission, absorption, scattering) are occurring.

- The equation of radiative transfer, which governs how radiation flows through things like stellar atmospheres, can be integrated over all solid angles to produce a **photon energy conservation equation** that looks a lot like others we've seen:

$$\oint d\Omega \left\{ \frac{1}{c}\frac{\partial I_\nu}{\partial t} + \hat{\mathbf{n}}\cdot\nabla I_\nu = j_\nu - \chi_\nu I_\nu \right\}$$

$$\frac{\partial U_\nu}{\partial t} + \nabla\cdot\mathbf{F}_\nu = 4\pi\chi_\nu(S_\nu - J_\nu)$$

If $S_\nu > J_\nu$ (i.e., RHS > 0), net emission of photons → particles **lose** thermal energy.

If $S_\nu < J_\nu$ (i.e., RHS < 0), net absorption of photons → particles **gain** thermal energy.

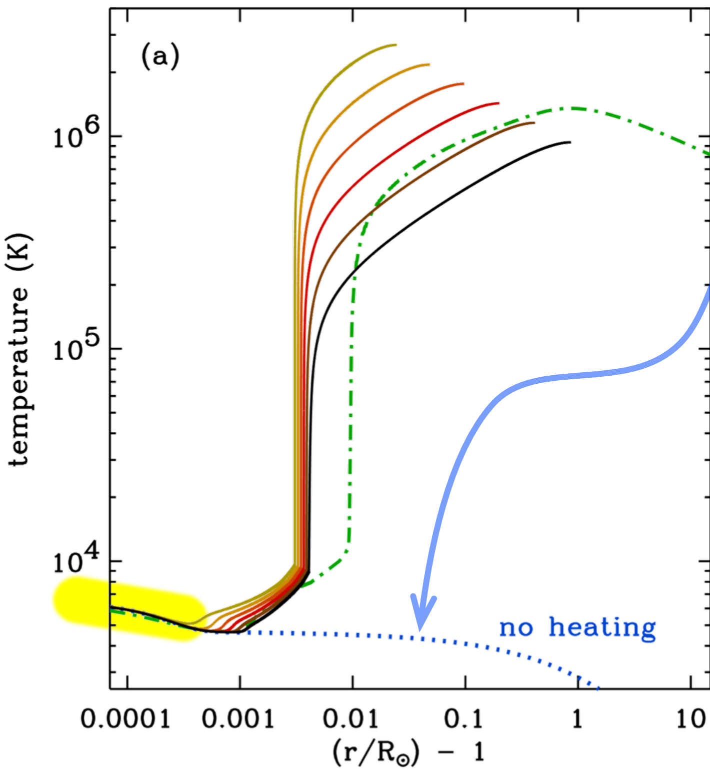
Thus,

$$Q_{\rm rad} = \int d\nu\, 4\pi\chi_\nu(J_\nu - S_\nu) = 4\pi\chi_{\rm eff}(J - S)$$

(2b) *Photospheric energy balance*

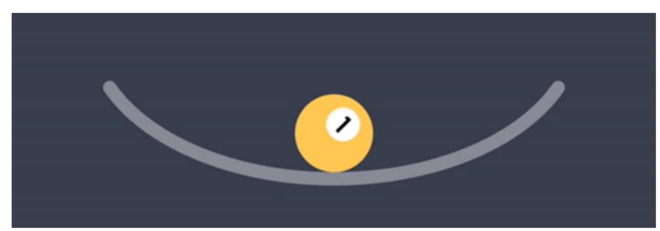
$$Q_{\rm rad} = \int d\nu \; 4\pi\chi_\nu (J_\nu - S_\nu) = 4\pi\chi_{\rm eff}(J - S) \propto T_{\rm rad}(\tau)^4 - T^4$$

- In local thermodynamic equilibrium (LTE), $S \approx B = \sigma_{\rm B} T^4/\pi$ and the mean intensity ends up as a function of optical depth in the atmosphere.
- When it's time-steady, $Q_{\rm rad} = 0$, and this means that $T = T_{\rm rad}$ **("radiative equilibrium").**



Note that it's a ***stable*** equilibrium…

- Increase T by a bit, $Q_{\rm rad}$ goes negative. The resulting cooling brings T back down.
- Decrease T by a bit, $Q_{\rm rad}$ becomes positive. The resulting heating brings T back up.



(2c) *Chromospheric energy balance*

- About 500 km above the photosphere, we see T start to increase.
- It's now a balance between $Q_{\rm rad}$ (which is negative because $T > T_{\rm rad}$) and $Q_{\rm heat}$.
- At these heights, the atmosphere is not LTE… it's **optically thin...** i.e., every new photon produced can be assumed to escape immediately.
- When $T >> T_{\rm rad}$, the rate depends on the atomic physics of dozens of elements (millions of transitions):

$$Q_{\rm rad} = 4\chi_{\rm eff}\sigma_{\rm B} \left(T_{\rm rad}^4 - T^4\right)$$

$$= -4\chi_{\rm eff}\sigma_{\rm B}T^4 \left(1 - \frac{T_{\rm rad}^4}{T^4}\right)$$

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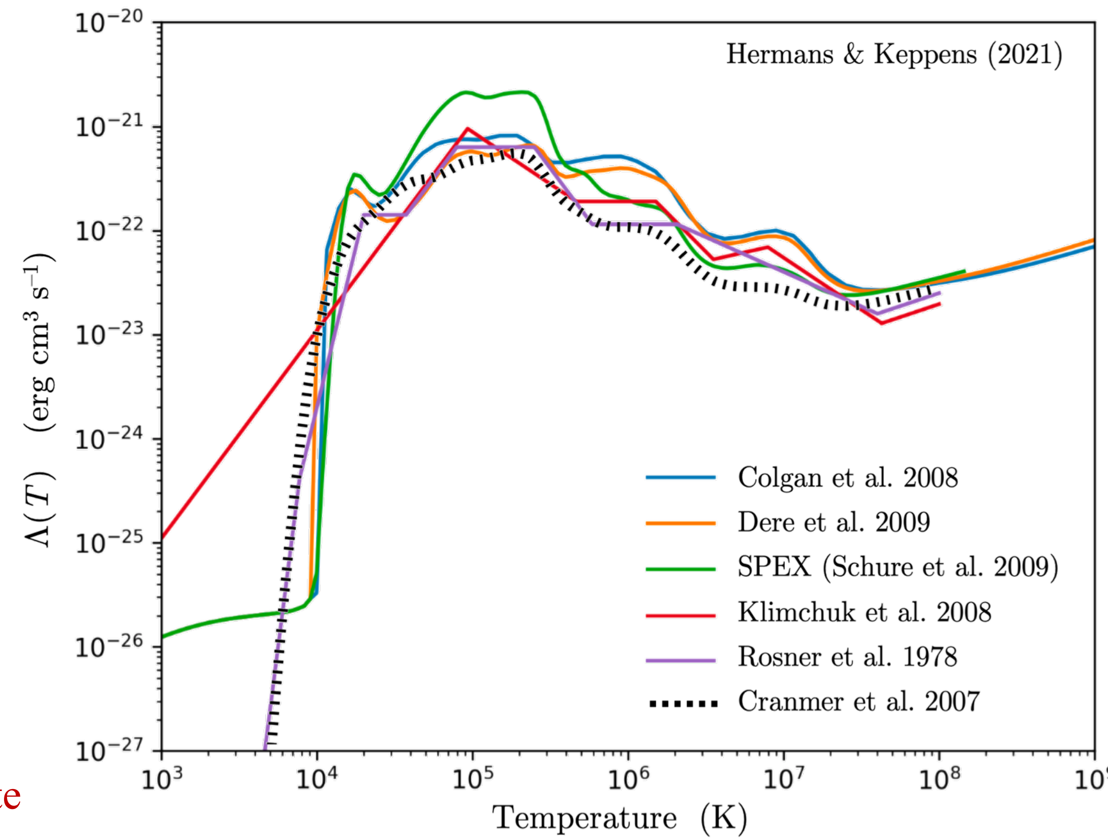
$$= -4\chi_{\rm eff}\sigma_{\rm B}T^4\underbrace{\left(1-\frac{T_{\rm rad}^4}{T^4}\right)}_{\approx 1}$$

$$= -\rho^2\,\Lambda(T)$$

$\Lambda(T)$ = optically thin radiative loss rate



(2c) *Chromospheric energy balance*

It's still not yet hot enough for heat conduction to be important, so in steady state…

$$Q_{\rm rad} + Q_{\rm heat} = 0 \qquad \rightsquigarrow \rightsquigarrow \qquad -\rho^2 \Lambda(T) + C\rho^{m/2} = 0$$

and we can write

$$\Lambda(T) = C\rho^{(m/2)-2}$$

which means that if m is between –1 and 2, then $\Lambda(T) \propto \rho^{-2.5}$ to ρ^{-1}

and that as we go up in height, density decreases, and thus $\rho^{(m/2)-2}$ must always be *increasing* as we go up.

(2c) *Chromospheric energy balance*

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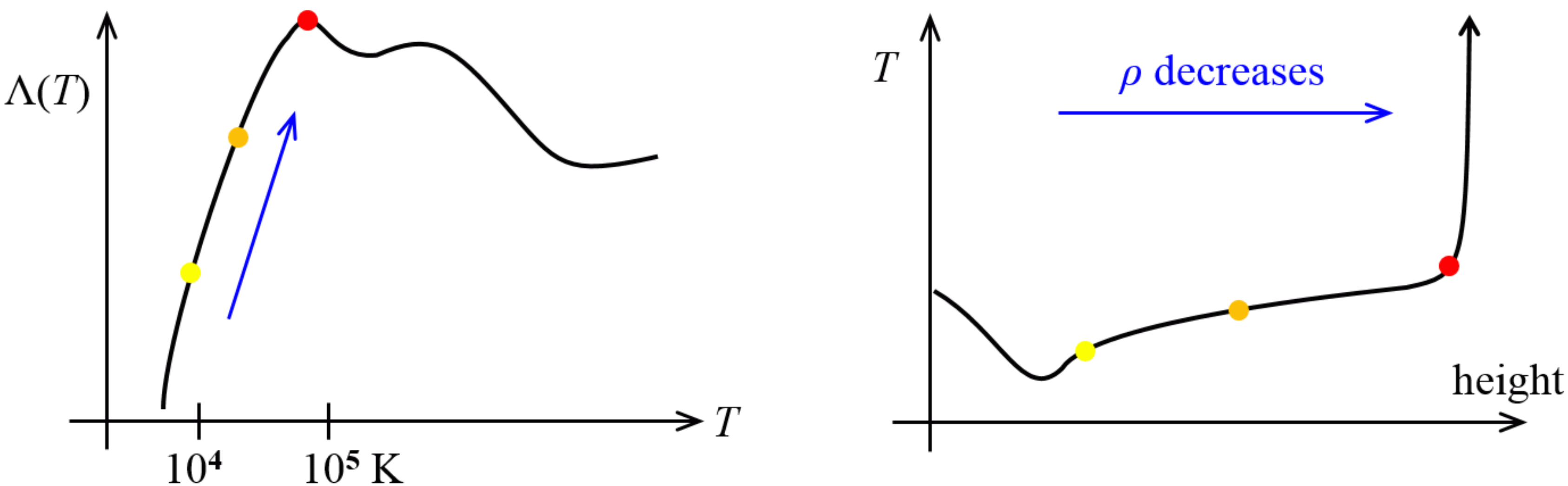
$$Q_{\rm rad} + Q_{\rm heat} = 0 \qquad \rightsquigarrow \rightsquigarrow \qquad -\rho^2 \Lambda(T) + C\rho^{m/2} = 0$$

and we can write

$$\Lambda(T) = C\rho^{(m/2)-2}$$

which means that if m is between –1 and 2, then $\Lambda(T) \propto \rho^{-2.5}$ to ρ^{-1}

and that as we go up in height, density decreases, and thus $\rho^{(m/2)-2}$ must always be *increasing* as we go up. Thus, **$\Lambda(T)$ must be increasing** as we go up, too…



(2c) *Chromospheric energy balance*

It's still not yet hot enough for heat conduction to be important, so in steady state…

$$Q_{\rm rad} + Q_{\rm heat} = 0 \quad \rightsquigarrow \rightsquigarrow \quad -\rho^2 \Lambda(T) + C\rho^{m/2} = 0$$

and we can write

$$\Lambda(T) = C\rho^{(m/2)-2}$$

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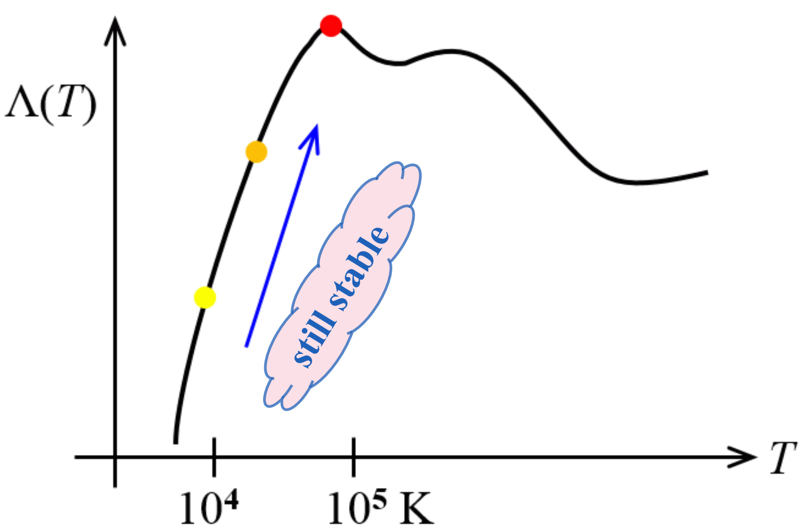
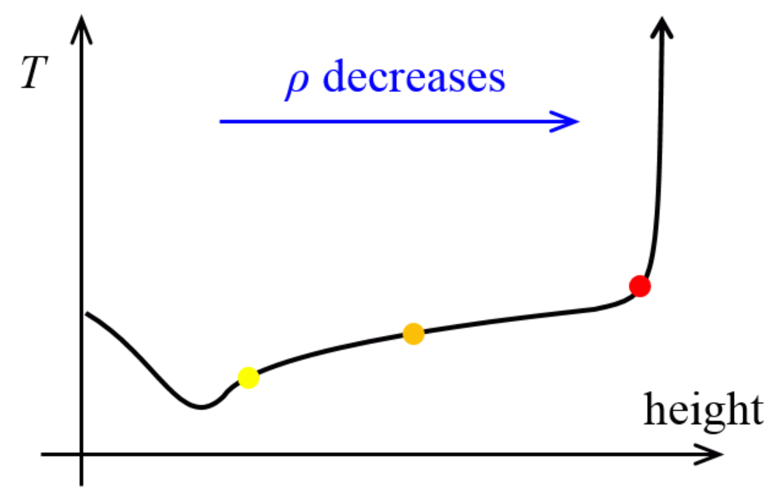
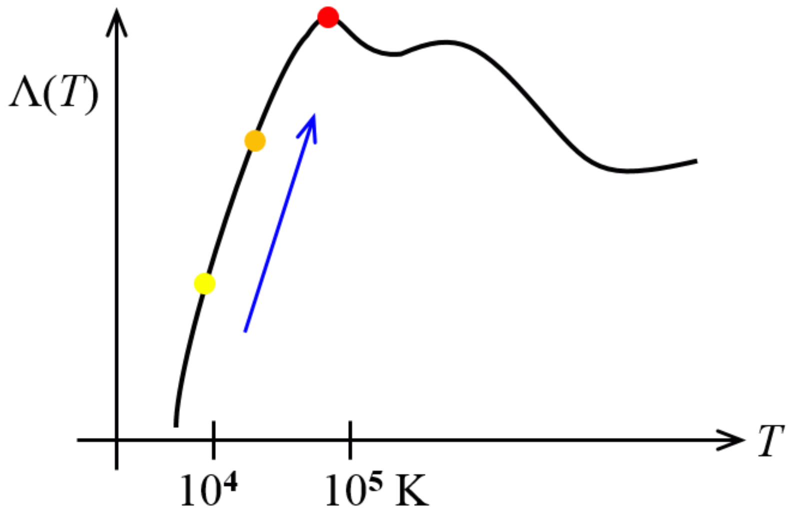
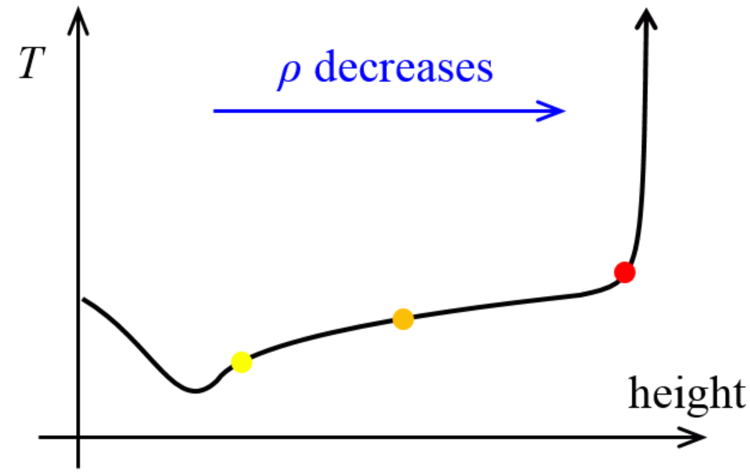
and that as we go up in height, density decreases, and thus $\rho^{(m/2)-2}$ must always be *increasing* as we go up. Thus, **Λ(*T*) must be increasing** as we go up, too…





(2c) *Chromospheric energy balance*





- At some point, *T* increases to the **peak** of the radiative loss curve around 100,000 K.
- If the plasma continues to be heated beyond that point, it can no longer find a stable equilibrium!
- This is why there's such a rapid/sharp **transition region** to coronal temperatures.
- There's a new equilibrium…

When Λ is decreasing as a function of T, it's **unstable:**

increase T

$\Downarrow$

$\Lambda\downarrow$, so less cooling

$\Downarrow$

T keeps increasing

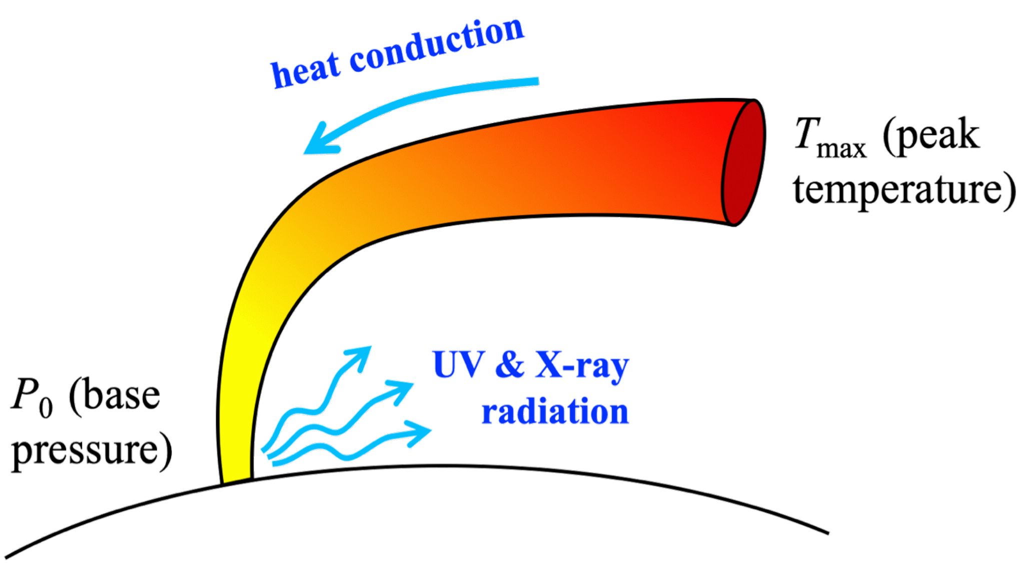
(2d) *Coronal energy balance*

- Once we get to temperatures $\gtrsim 10^6$ K, heat conduction becomes an important contributor.
- In the transition region, it's a balance between 3 roughly-equal pieces:

$$Q_{\rm rad} + Q_{\rm heat} + Q_{\rm cond} = 0$$

and the solution method was first worked out by Rosner, Tucker, & Vaiana (1978) **(RTV).**

- We'll examine this in more detail in Paper 2 and the hands-on exercise.



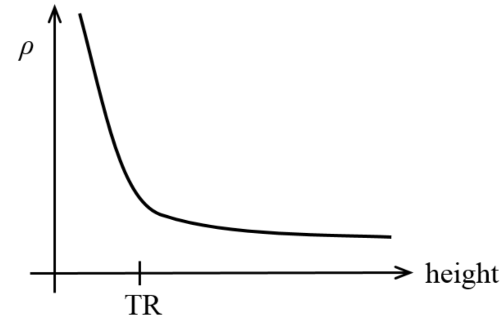
But if we go up just a bit into the low corona, $Q_{\rm rad}$ becomes much less important:

$$Q_{\rm heat} + Q_{\rm cond} \approx 0$$

so, for now, we'll assume this as we solve for the coronal $T(r)$.

(2d) *Coronal energy balance*

- One more simplification: let's assume $\rho \approx$ constant in the low corona. When T gets large, the hydrostatic scale height $H \propto kT/mg$ becomes large.
- Thus, it's not terrible to assume $Q_{\rm heat} \approx$ constant, too.



- Using Cartesian coordinates because we're close to the solar surface our energy balance equation is

$$Q_{\rm heat} = -K_0 \frac{d}{dz}\left(T^{5/2}\frac{dT}{dz}\right)$$

Let's say we're looking at a coronal loop, with maximum height L. We know that $T(z)$ increases in the corona, and we can reasonably guess that $T = T_{\rm max}$ at $z = L$. If we define new variables,

$$x = \frac{z}{L} \qquad \text{and} \qquad y = \left(\frac{T}{T_{\rm max}}\right)^{7/2}$$

then the differential equation becomes

$$\frac{d^2y}{dx^2} = -\frac{7L^2 Q_{\rm heat}}{2K_0 T_{\rm max}^{7/2}} \equiv -\xi = \{\text{constant}\} \ .$$

(2d) *Coronal energy balance*

- Before we solve the differential equation, let's think about its **boundary conditions…**

$$x = \frac{z}{L} \qquad \text{and} \qquad y = \left(\frac{T}{T_{\rm max}}\right)^{7/2}$$

$$\frac{d^2y}{dx^2} = -\frac{7L^2 Q_{\rm heat}}{2K_0 T_{\rm max}^{7/2}} \equiv -\xi = \{\text{constant}\}$$

1. $y(1) = 1$ (at the top of the loop, we've defined T to be at its maximum value)
2. $y'(1) = 0$ (there should be symmetry at the top, with T reaching a smooth extremum)
3. $y(0) \approx 0$ (note that $(10^4/10^6)^{7/2} \approx 10^{-7}$, which is pretty close to zero)

Wait… how many boundary conditions do we *need*…?

(2d) *Coronal energy balance*

$$\frac{d^2y}{dx^2} = -\frac{7L^2Q_{\rm heat}}{2K_0T_{\rm max}^{7/2}} \equiv -\xi = \{\rm constant\} .$$

General solution: $y(x) = -\frac{1}{2}\xi x^2 + C_1 x + C_2.$

(2d) *Coronal energy balance*

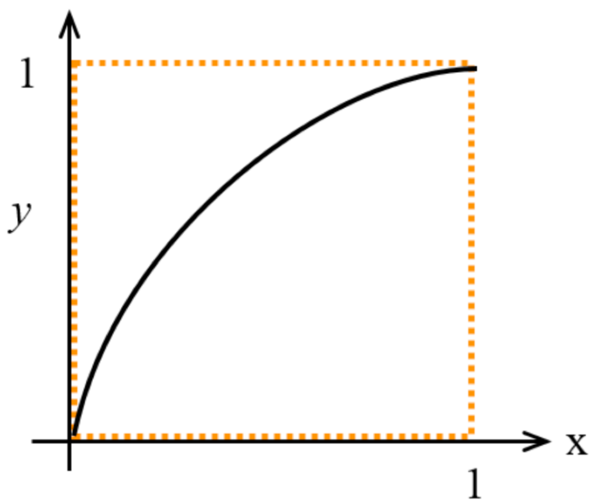
$$\frac{d^2y}{dx^2} = -\frac{7L^2 Q_{\rm heat}}{2K_0 T_{\rm max}^{7/2}} \equiv -\xi = \{\rm constant\} \ .$$

General solution: $y(x) = -\frac{1}{2}\xi x^2 + C_1 x + C_2$.

There are some assumptions to make about boundary conditions, and those conditions tell us that ξ must be ≈ 2. With that, the solution for $y(x)$ is a piece of a *concave-down parabola:*

$$y(x) = 1 - (x-1)^2$$

$y(0) \approx 0$
$y(1) \approx 1$
$y'(1) \approx 0$



(2d) *Coronal energy balance*

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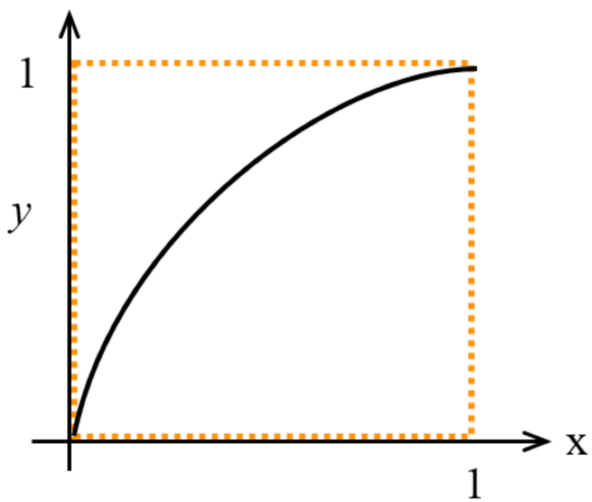
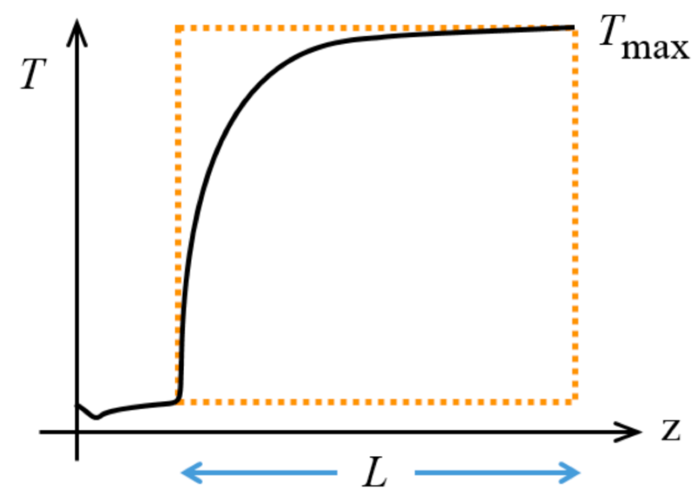
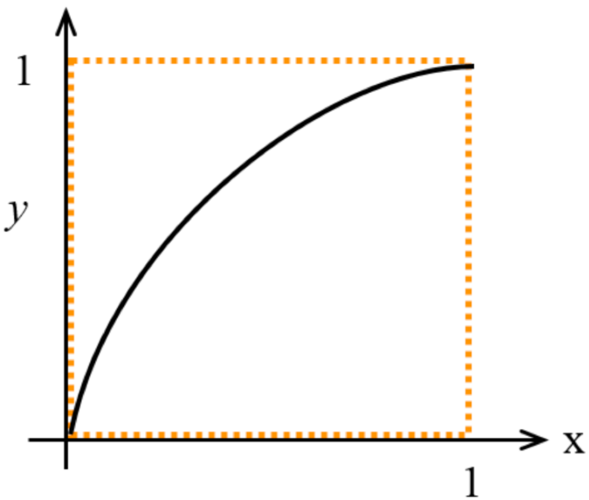
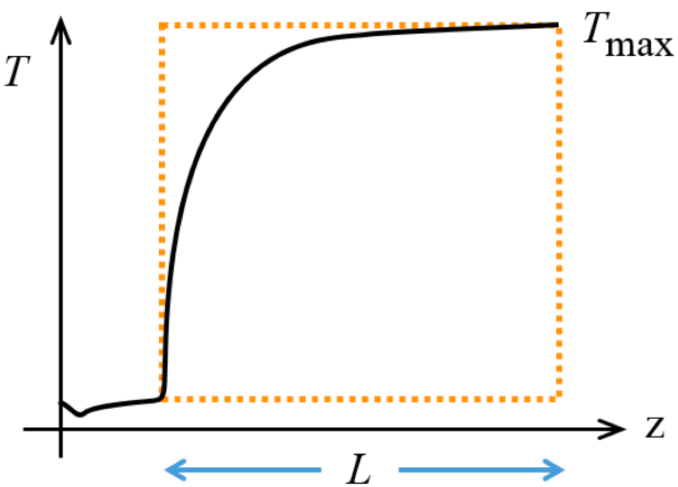




(2d) *Coronal energy balance*





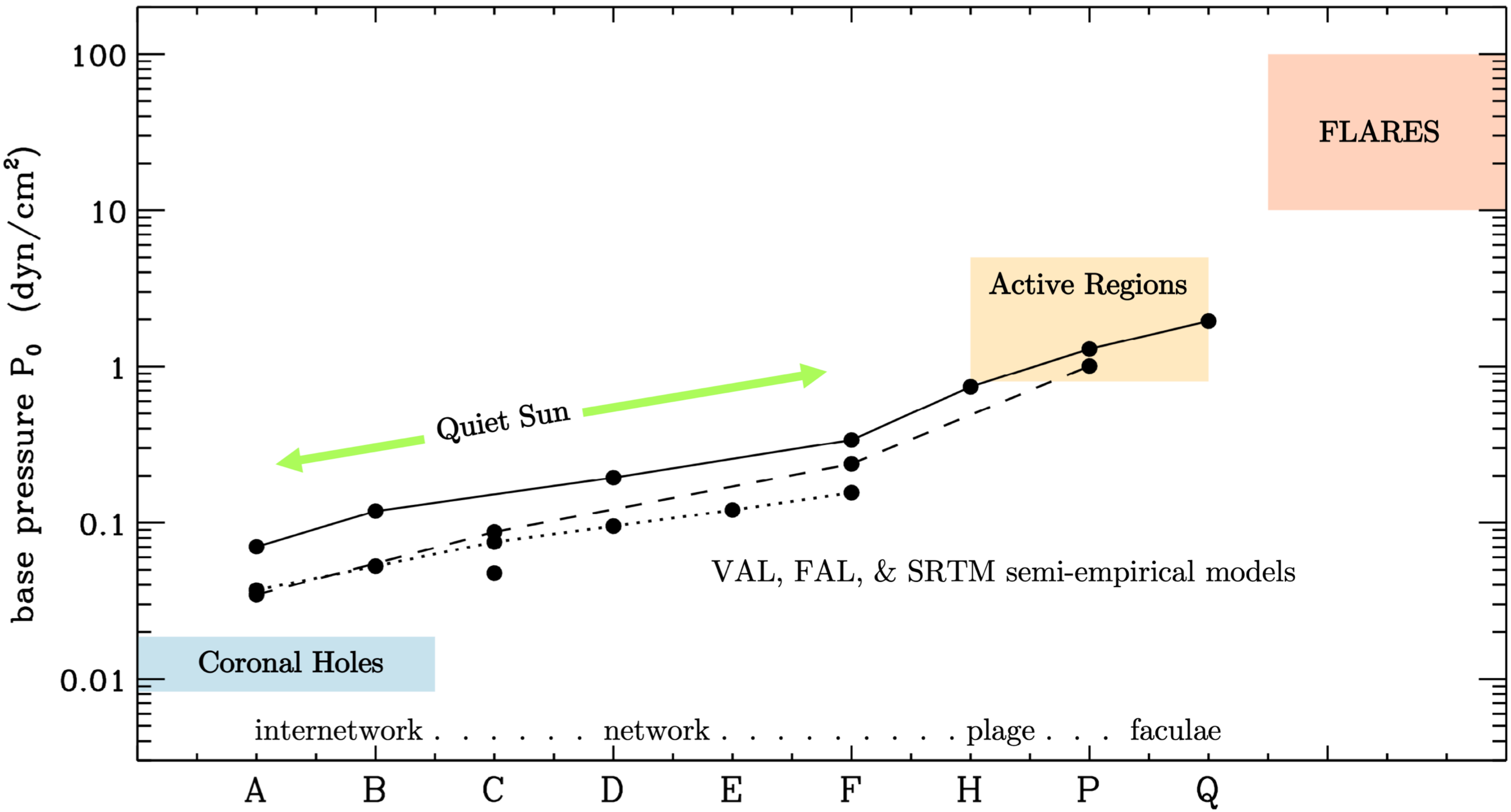
- Because *dy/dx* ∝ *x,* near the base the conduction flux ($T^{5/2}\, dT/dz$) ~ constant, so as *T* drops toward the chromosphere, *dT/dz* must get steeper.
- Thus, there are 2 reasons why the TR is so sharp: one bottom-up, one top-down!
- Lastly, $\xi = 2$ implies that

$$T_{\rm max} = \left(\frac{7}{4K_0}\right)^{2/7} Q_{\rm heat}^{2/7}\ L^{4/7}$$

Double the heating, and $T_{\rm max}$ increases by only a factor of $2^{2/7} \approx 1.22$.
Conduction acts as a **"thermostat"** that smooths out the impact of coronal heating.

(2d) *Coronal energy balance*

- My model didn't provide the base pressure (or density) that tells us how much plasma fills the loop. Full-on RTV-type models do. They help us understand the observed values:



(2e) *Outer corona & heliosphere*

	(2b) Photosphere	(2c) Chromosphere	Transition Region	(2d) Inner Corona	(2e) Outer Corona & Wind
$Q_{\rm rad}$	🌞	🌞	🌞		
$Q_{\rm heat}$		🌞	🌞	🌞	?
$Q_{\rm cond}$			🌞	🌞	🌞
$Q_{\rm adv}$				?	?

- We'll start considering the boxes with "?" when we discuss the solar wind.
- For now, let's just try to understand Sidney Chapman's (1957) derivation of how $T(r)$ in the outer corona ought to behave if…

$$Q_{\rm cond} \approx 0$$

(2e) *Outer corona & heliosphere*

- If the outer corona is **spherically symmetric,** then

$$Q_{\rm cond} \;=\; -\nabla\cdot\mathbf{q}_{\rm cond} \;=\; \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 K_0\, T^{5/2}\,\frac{\partial T}{\partial r}\right) \;=\; 0$$

and thus,

$$r^2\, T^{5/2}\,\frac{dT}{dr} \;=\; \text{constant}, \qquad \text{so} \qquad T^{5/2}\, dT \;=\; C\,\frac{dr}{r^2}$$

- Integrating, with boundary condition that $T \rightarrow 0$ as $r \rightarrow \infty$, we get that $T(r) \propto r^{-2/7}$.

(2e) *Outer corona & heliosphere*

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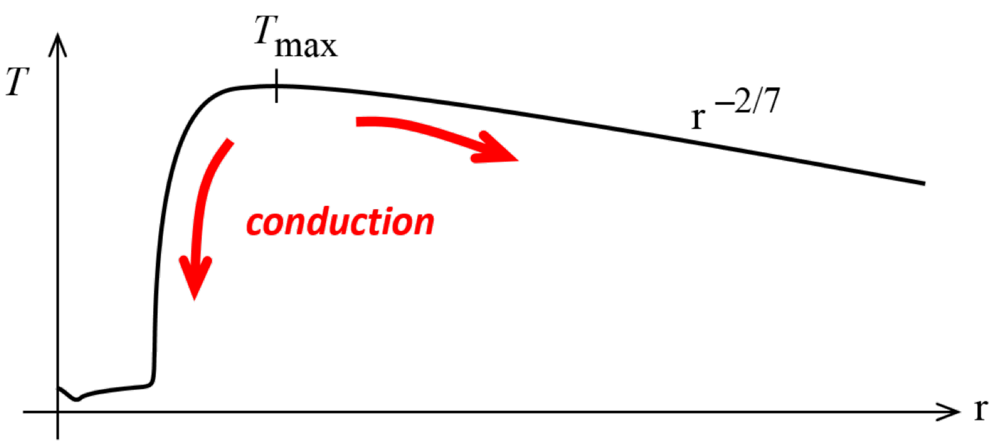
$$Q_{\rm cond} = -\nabla \cdot \mathbf{q}_{\rm cond} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 K_0 T^{5/2} \frac{\partial T}{\partial r}\right) = 0$$

and thus,

$$r^2 T^{5/2} \frac{dT}{dr} = \text{constant}, \qquad \text{so} \qquad T^{5/2}\, dT = C \frac{dr}{r^2}$$

- Integrating, with boundary condition that $T \to 0$ as $r \to \infty$, we get that $T(r) \propto r^{-2/7}$.
- In other words, heat is "deposited" at some height in the low corona, and it gets conducted "out" (i.e., both up and down) from there…



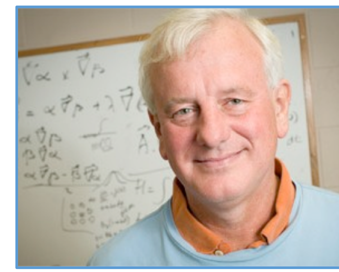
Again we see the **"thermostat"** effect… conduction smears out the thermal energy and creates a flatter $T(r)$ than would otherwise occur (say, from adiabatic parcels).

For next week

- If we didn't make it all the way through these slides, please read them all.

- We'll start engaging with an extension of the RTV (1978) model that accounts for heating, radiation, and conduction… for heating rates & magnetic fields that vary with height.

- Read paper 2: "Scaling laws and temperature profiles for solar and stellar coronal loops with non-uniform heating" (2010, ApJ, 714, 1290), by Piet Martens https://ui.adsabs.harvard.edu/abs/2010ApJ...714.1290M/abstract



- If pressed for time… sections to **read** vs. **skip:**

  1, 2, 2.1, 2.2, 3, 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 4, appendix

- Engage with the Slack discussion

- Be ready to run python Jupyter notebooks, either in the cloud (https://jupyter.org/) or on a local computer…