#### **ASTR-5540: MATHEMATICAL METHODS**

CU Boulder Syllabus (Fall 2024)

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Course Times: Mon., Wed., Fri., 10:10–11:00 am, Duane Physics room E126 https://stevencranmer.bitbucket.io/ASTR\_5540\_2024/

**Office Hours:** TBD dates, times, & modes (in-person or virtual)

### **SUMMARY**

This is an applied mathematics course designed to provide the necessary analytical and numerical background for courses in astrophysics, planetary science, plasma physics, fluid dynamics, electromagnetism, and radiation transfer. Topics include linear algebra, integration techniques, ordinary and partial differential equations, special functions, integral transforms, and integral equations. We aim to keep the course grounded in research applications and illustrative examples from the topical areas listed above. This is a core required course for APS graduate students, and it is the same course as ATOC–5540.

### **COURSE MATERIAL**

The primary "required readings" are the lecture notes, which ought to contain everything discussed in class. They will be posted on both Canvas and the course web page as the semester progresses. No one textbook covers all aspects of this class, but here are several good ones that I can recommend:

- *Mathematical Methods for Physicists*, by Arfken, Weber, & Harris, 7th ed., 2013. A useful resource for your physics bookshelf. [online from CU domains]
- *Mathematical Methods for Physics and Engineering*, by Riley, Hobson, & Bence, 3rd ed., 2006. The traditional textbook for many past iterations of this course.
- Mathematical Methods of Physics, by Mathews & Walker, 2nd ed., 1970. Old, but classic source of next-level analytic tricks.
- *Numerical Recipes: The Art of Scientific Computing*, by Press, Teukolsky, Vetterling, & Flannery, 3rd ed., 2007. It's where I learned numerical methods, and I refer back to it often. [online]
- Numerical Methods that (Usually) Work, by Acton, 2nd ed., 1990. Also old, but full of insight and irreverent entertainment. Like some professors I know?

There will also be many more links to additional online material on this course's web page. Please see me if you have difficulty obtaining any of the recommended materials in convenient formats.

## **GRADING**

The final course grade will be assembled from the following components:

Homework (Problem Sets)	_
Midterm Project (Choose Your Own Adventure)	
Class Participation	
Final Exam (Take-Home)	

Each graded aspect of this course is described in more detail below. APS graduate students need to earn a grade of B— or higher in this course to meet the *Preliminary Core-Course* requirement (i.e., before moving on to take the Comprehensive Exam for Ph.D. candidacy). If a grade lower than B— is received, there are a variety of other options, including taking the course again or arranging an independent study. For more details, please contact the APS Office or read the preliminary exam policy.

### SCHEDULE OF TOPICS

The dates listed here for each set of topics are approximate. The schedule on the web page will be kept up-to-date, and we will also maintain a "daily topic log" on Canvas that describes exactly what was covered on each day.

Introduction & overview	Aug 26
1. Algebraic techniques, computing, & root finding	Aug 26, 28, 30; Sep 4
2. Linear algebra (focus on useful applications)	Sep 6, 9, 11, 13, 16, 18
3. Integrals, numerical quadrature, & special functions	Sep 20, 23, 25, 27
4. Ordinary differential equations (analytic methods)	Sep 30; Oct 2, 4, 7, 9, 11, 14
5. Ordinary differential equations (numerical methods)	Oct 16, 18, 21, 23
6. Integral transforms & the discrete Fourier transform	Oct 25, 28, 30; Nov 1, 4
7. Partial differential equations (analytic methods)	Nov 6, 8, 11, 13, 15, 18, 20, 22
8. Partial differential equations (numerical methods)	Dec 2, 4, 6
9. Integral & integro-differential equations	Dec 9, 11

#### **HOMEWORK**

There will be six problem-set homework assignments. Some problems will focus on analytic/paper-and-pencil work, and others will involve coding. For the latter, feel free to use whatever programming languages or packages you're most comfortable with. On this course's web page, I will post some links to resources and tutorials for scientific computing with Python.

Submitting homeworks either on paper or electronically (on Canvas) is fine, though if you choose the latter, please compile your submission into a single PDF. When grading, I usually have each numbered problem be worth 10 points. Thus, if a given homework set has 4 problems, the grade will probably be listed in Canvas as out of 40 points. Only at the end of the semester will we "renormalize" each homework set to be worth (55% / 6) = 9.16667% of the total course grade.

Problems will be due on the dates to be listed, but one late submission can be arranged if necessary (for a maximum delay of 1 week), as long as the arrangement is made at least 1 class prior to the due date. Other late problem sets usually incur a penalty of a 5% lower grade per weekday that it is late. Submissions are no longer possible after answer keys are distributed (usually when graded homeworks are returned, about 1 week after the due date).

# **MIDTERM PROJECT**

Some time around Week 10 of the semester, the usual two-week cadence of homework assignments will be interrupted by a different activity. For this, you can choose one of three options:

- You can create and write up two new homework problems (with answer keys) of similar size and scope as the existing homework problems for this course. Please consider this option if I have not yet shown any examples or applications from the topical research area that you find most interesting!
- You can choose a relevant paper (from a journal, or arXiv, or references that I give in the lecture notes) and review it with two written components: (a) a brief pedagogical summary, as if you were preparing lecture notes to teach it to a new class of Math Methods students, and (b) a mock referee's report, which includes criticism for the authors that is as specific and constructive as possible.
- You can choose a relevant paper (as above) and reproduce some of its key results. This is likely to involve some combination of writing code, re-deriving equations, downloading data, and making

figures. Your write-up will contain your versions of the results as well as insights about how straightforwardly "reproducible" the original paper was.

More information about these options will be provided soon. A week or two prior to the time set aside for this activity, you will be asked to submit details about which option you will pursue.

#### **CLASS PARTICIPATION**

For me, a perpetually unsolved problem is "How does one assign grades for in-class engagement?" for classes that do not use clickers or other straightforward ways of quantifying participation. I'm never going to take attendance in a graduate course, and it would be impossible (and a bit unfair) to count the number of times students raise their hands to ask questions in class. My current favorite solution to this problem is the following smörgåsbord approach, in which you can gather participation credit in a number of different ways. Each time you do any of the following things, you'll get one point:

- Come to Office Hours with specific course-relevant things to ask or chat about.
- Post a question (or any course-relevant thoughts) to the Canvas Discussion Board, or answer someone else's question in that forum.
- Ask a question in class that requires me to go off and do some extra research to determine the answer.
- Actively work on one of the few in-class exercises that we will do (the dates of which I'll try to announce well in advance).

Repetitions in any category are allowed. Accumulate 5 points, and you've got full participation credit.

#### FINAL TAKE-HOME EXAM

The APS core graduate courses are required to include an individual final examination that broadly samples the course material. This semester, there will be a take-home exam with problems having a similar scope as the homework problem sets; the primary difference being the **no-collaboration** aspect of an exam. The exact dates will be given on the course web page and Canvas, but I anticipate that the exams will be due approximately 5 to 7 days after they are distributed.

### ADDITIONAL GRADING NOTES

At the end of the semester, numerical scores from each of the four primary components will be summed up. Also, to account for the overall challenge level of the course, I usually add some number of extra (percentage) points to everyone's course totals. For example, some homework problems were honed from ones used in earlier courses, but many have been crafted for the first time. Thus, these extra points are a kind of compensation for you being "guinea pigs" on the brand-new subset of the coursework. Although this is not really a "curve" (because it doesn't depend on the relative distribution of grades) it does attempt to provide a corrective to aspects that may have been overly difficult. The total number of percentage points to be added is not likely to be larger than 3 or 4. Once the extra points have been added, the conversion to letter grades will be done using the following standard CU system:

	<b>A</b> (93 and up)	<b>A-</b> (90 to 93)
<b>B+</b> (87 to 90)	<b>B</b> (83 to 87)	<b>B</b> – (80 to 83)
<b>C+</b> (77 to 80)	<b>C</b> (73 to 77)	<b>C</b> – (70 to 73)
<b>D+</b> (67 to 70)	<b>D</b> (63 to 67)	<b>D</b> – (60 to 63)
<b>F</b> (below 60)		

Please feel free to email, drop by Office Hours, or just chat informally after class, if you have questions.

#### ACADEMIC INTEGRITY

All students enrolled in a University of Colorado Boulder course are responsible for knowing and adhering to the Honor Code. Violations of the Honor Code may include, but are not limited to: plagiarism (including use of paper writing services or technology [such as essay bots]), cheating, fabrication, lying, bribery, threat, unauthorized access to academic materials, clicker fraud, submitting the same or similar work in more than one course without permission from all course instructors involved, and aiding academic dishonesty. All incidents of academic misconduct will be reported to Student Conduct & Conflict Resolution (honor@colorado.edu, 303-492-5550). Students found responsible for violating the Honor Code will be assigned resolution outcomes from Student Conduct & Conflict Resolution as well as be subject to academic sanctions from the faculty member. Visit Honor Code for more information on the academic integrity policy.

For this course, I encourage you to discuss the assignments and topics with your fellow students. However, everything that is written up and submitted must be your own independent work. If you do collaborate with other students, a good time to split off from the group is when you start to write up your answers. If someone were to ask you questions about your work, you should be able to explain everything about how & why you did it the way you did. The APS Department also provides some additional guidance on coursework collaboration.

### **CLASSROOM BEHAVIOR**

Students and faculty are responsible for maintaining an appropriate learning environment in all instructional settings, whether in person, remote, or online. Failure to adhere to such behavioral standards may be subject to discipline. Professional courtesy and sensitivity are especially important with respect to individuals and topics dealing with race, color, national origin, sex, pregnancy, age, disability, creed, religion, sexual orientation, gender identity, gender expression, veteran status, political affiliation or political philosophy. For more information, see the classroom behavior policy, the Student Code of Conduct, and the Office of Institutional Equity and Compliance.

The policy of the Department of Astrophysical and Planetary Sciences is to ban any use of electronic devices (cellphones, tablets, laptops) in class except as an approved accommodation granted by Disability Services, or as explicitly authorized by the instructor. *In this course* I authorize the use of tablets and laptops for note-taking, but in-person students doing so must do their best to seat themselves with nobody behind them.

# REQUIREMENTS FOR INFECTIOUS DISEASES

Members of the CU Boulder community and visitors to campus must follow university, department, and building health and safety requirements and all public health orders to reduce the risk of spreading infectious diseases.

The CU Boulder campus is currently mask optional. However, if masks are again required in classrooms, students who fail to adhere to masking requirements will be asked to leave class. Students who do not leave class when asked or who refuse to comply with these requirements will be referred to Student Conduct & Conflict Resolution. Students who require accommodation because a disability prevents them from fulfilling safety measures related to infectious disease will be asked to follow the steps in the "Accommodation for Disabilities" statement on this syllabus.

For those who feel ill and think you might have COVID-19, or if you have tested positive for COVID-19, please stay home and follow the further guidance of the Public Health Office. For those who have been in close contact with someone who has COVID-19 but do not have any symptoms and have not tested positive for COVID-19, you do not need to stay home.

#### ACCOMMODATION FOR DISABILITIES

Disability Services determines accommodations based on documented disabilities in the academic environment. If you qualify for accommodations because of a disability, submit your accommodation letter from Disability Services to your faculty member in a timely manner so your needs can be addressed. Contact Disability Services at 303-492-8671 or dsinfo@colorado.edu for further assistance. If you have a temporary medical condition or injury, see the guidelines for Temporary Medical Conditions on the Disability Services website.

#### PREFERRED STUDENT NAMES AND PRONOUNS

CU Boulder recognizes that students' legal information doesn't always align with how they identify. Students may update their preferred names and pronouns via the student portal; those preferred names and pronouns are listed on instructors' class rosters. In the absence of such updates, the name that appears on the class roster is the student's legal name.

# MISCONDUCT, DISCRIMINATION, HARASSMENT, AND/OR RETALIATION

CU Boulder is committed to fostering an inclusive and welcoming learning, working, and living environment. University policy prohibits protected-class discrimination and harassment, sexual misconduct (harassment, exploitation, and assault), intimate partner violence (dating or domestic violence), stalking, and related retaliation by or against members of our community on- and off-campus. These behaviors harm individuals and our community. The Office of Institutional Equity and Compliance (OIEC) addresses these concerns, and individuals who believe they have been subjected to misconduct can contact OIEC at 303-492-2127 or email cureport@colorado.edu. Information about university policies, reporting options, and other support resources can be found on the OIEC website.

Please know that faculty and instructors have a responsibility to inform OIEC when they are made aware of incidents related to these policies regardless of when or where something occurred. This is to ensure that individuals impacted receive an outreach from OIEC about their options for addressing a concern and the support resources available. To learn more about reporting and support resources for a variety of issues, visit Don't Ignore It.

# **RELIGIOUS OBSERVANCES**

Campus policy regarding religious observances requires that faculty make every effort to deal reasonably and fairly with all students who, because of religious obligations, have conflicts with scheduled exams, assignments, or required attendance. For full details, see the campus policy regarding religious observances.

### MENTAL HEALTH AND WELLNESS

The University of Colorado Boulder is committed to the well-being of all students. If you are struggling with personal stressors, mental health or substance use concerns that are impacting academic or daily life, please contact Counseling and Psychiatric Services (CAPS) located in C4C or call 303-492-2277. Free and unlimited telehealth is also available through Academic Live Care. The Academic Live Care site also provides information about additional wellness services on campus that are available to students.

I try to provide a positive and supportive learning environment for everyone, and it's always helpful for me to hear what works best for you. Have a great semester!