ASTR–5540: MATHEMATICAL METHODS

CU Boulder Syllabus (Fall 2022)

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Course Times: Mon., Wed., Fri., 10:10–11:00 am, Duane Physics room E126

Course Web Page: https://stevencranmer.bitbucket.io/ASTR_5540_2022/

Office Hours: TBD dates, times, & modes (in-person or virtual)

SUMMARY

This is an applied mathematics course designed to provide the necessary analytical and numerical background for courses in astrophysics, planetary science, plasma physics, fluid dynamics, electromagnetism, and radiation transfer. Topics include linear algebra, integration techniques, ordinary and partial differential equations, special functions, integral transforms, and integral equations. We aim to keep the course grounded in research applications and illustrative examples from the areas of physics listed above. This is a core required course for APS graduate students, and it is the same course as ATOC–5540.

COURSE MATERIAL

The primary "required readings" are the lecture notes, which ought to contain everything discussed in class. They will be posted on the course web page as the semester progresses. No one textbook covers all aspects of this class, but here are several good ones that I can recommend:

- *Mathematical Methods for Physicists,* by Arfken, Weber, & Harris, 7th ed., 2013. A useful resource for your physics bookshelf. [online from CU domains]
- *Mathematical Methods for Physics and Engineering,* by Riley, Hobson, & Bence, 3rd ed., 2006. The traditional textbook for many past iterations of this course.
- *Mathematical Methods of Physics,* by Mathews & Walker, 2nd ed., 1970. Old, but classic source of next-level analytic tricks.
- *Numerical Recipes: The Art of Scientific Computing,* by Press, Teukolsky, Vetterling, & Flannery, 3rd ed., 2007. It's where I learned numerical methods, and I refer back to it often. [online]
- *Numerical Methods that (Usually) Work,* by Acton, 2nd ed., 1990. Also old, but full of insight and irreverent entertainment. Like some professors I know?

There will also be additional links to additional online material on this course's web page. Please see me if you have difficulty obtaining any of the recommended materials in convenient formats.

GRADING

The final course grade will be assembled from the following components:

Homework Problem Sets 1–6	50%
Take-Home Midterm Exam	15%
Participation in Class & Canvas Discussions	10%
Final Project	25%

Each graded aspect of this course is described in more detail below. APS graduate students need to earn a grade of B– or higher in this course to meet the *Preliminary Core-Course* requirement (i.e., before moving on to take the Comprehensive Exam for Ph.D. candidacy). If a grade lower than B– is received, there are a variety of other options, including taking the course again or arranging an independent study. For more details, please contact the APS Office or read the preliminary exam policy.

SCHEDULE OF TOPICS

The dates listed here for each set of topics are approximate. The web page will be kept up-to-date on the topics to be covered in each class session.

Introduction & overview	Aug 22
1. Algebraic techniques, computing, & root finding	Aug 24, 26, 29, 31
2. Linear algebra (focus on useful applications)	Sep 2, 7, 9, 12
3. Integrals, numerical quadrature, & special functions	Sep 14, 16, 19, 21, 23
4. Ordinary differential equations (analytic methods)	Sep 26, 28, 30; Oct 3, 5, 7, 10
5. Ordinary differential equations (numerical methods)	Oct 12, 14, 17, 19, 21
6. Integral transforms & the discrete Fourier transform	Oct 24, 26, 28, 31
7. Partial differential equations (analytic methods)	Nov 2, 4, 7, 9, 11, 14
8. Partial differential equations (numerical methods)	Nov 16, 18, 28, 30
9. Integral & integro-differential equations	Dec 2, 5, 7

CLASS PARTICIPATION

Once every one or two weeks (likely on Fridays) we will devote up to 20–30 minutes to an informal recitation/discussion session that will be led mostly by student feedback. To prepare, there will be ongoing Canvas discussion boards where you can respond to specific prompts. In those responses, you can ask questions about the lectures or homework, point out steps you didn't understand, correct any errors of mine, or recommend cool/new/relevant papers you've seen on arXiv. It is still not yet clear how many of these sessions—and thus how many Canvas discussion boards—there will be. If there end up being N sessions, you will earn full credit for this "participation" component of the course grade by submitting at least $2N$ individual responses to the discussion boards.

HOMEWORK

There will be five or six primary problem-set based homeworks, and they will count towards 50% of the course grade. Each problem set will be weighted equally. Some will focus on analytic/paper-and-pencil work, and others will involve coding. For the latter, feel free to use whatever programming languages or packages you're most comfortable with. On this course's web page, I will post some links to resources and tutorials for scientific computing with Python.

A detailed schedule of distribution and due dates will be posted on the web page. Either hardcopy or online (Canvas) submission is fine, though if you choose the latter, please compile your submission into a single PDF. Problems will be due on the dates listed, but one late submission can be arranged if necessary (for a maximum delay of 1 week), as long as the arrangement is made at least 1 class prior to the due date. Other late problem sets usually incur a penalty of a 5% lower grade per weekday that it is late. Submissions are no longer possible after answer keys are distributed (usually when graded homeworks are returned, about 1 week after the due date).

MIDTERM EXAM

The format will be a take-home exam with problems having a similar scope as the homework problem sets; the primary difference being the **no-collaboration** aspect of an exam. Completed exams will be due one week after they are given to you, and exact dates will be given on the course web page.

FINAL PROJECT

The APS core graduate courses are required to include an individual final examination that broadly samples the course material. Here, this will take the form of a project that will enable you to explore a chosen topic in a bit more detail and gain some extra experience with scientific writing and coding. The idea is to apply the concepts of this course to solving a specific problem or application (which should go beyond the actual material discussed in class). To ensure the "broad sampling of course material," we require that the project involve: (a) both analytic and numerical methods, and (b) two or more distinct mathematical techniques from the topics to be covered in the course.

In one of the homeworks in the middle of the semester, I'll be asking you to select a topic and to briefly describe what you intend to do for your project. I will then provide feedback and a formal go-ahead to proceed. (This is often necessary because some initial ideas may be overly ambitious or contain hidden pitfalls.) The projects will be graded on the basis of criteria such as:

- Providing a brief written description of the problem itself (e.g., motivation & background).
- Successful writing of numerical code & generation of a solution.
- Use of analytic methods to compare with, or validate, the numerical solutions.
- Clear & informative presentation of results through illustrations, tables, cartoons, etc.
- Providing a professional & concise written discussion of those results.

Additional information, including lists of example topic ideas and deadlines, will be distributed during the semester. Please feel free to discuss possible topics with the instructor at any time.

ACADEMIC INTEGRITY

All students enrolled in a University of Colorado Boulder course are responsible for knowing and adhering to the Honor Code academic integrity policy. Violations of the Honor Code may include, but are not limited to: plagiarism, cheating, fabrication, lying, bribery, threat, unauthorized access to academic materials, clicker fraud, submitting the same or similar work in more than one course without permission from all course instructors involved, and aiding academic dishonesty. All incidents of academic misconduct will be reported to the Honor Code (honor@colorado.edu; 303–492–5550). Students found responsible for violating the academic integrity policy will be subject to nonacademic sanctions from the Honor Code as well as academic sanctions from the faculty member. Additional information regarding the Honor Code academic integrity policy can be found at the Honor Code website.

For this course, I encourage you to discuss the assignments and topics with your fellow students. However, everything that is written up and submitted must be your own independent work. If you do collaborate with others, a good time to split off from the group is when you start to write up your answers. If someone were to ask you questions about your work, you should be able to explain everything about how & why you did it the way you did. For more information, see a related APS-specific Honor Code statement.

RELIGIOUS OBSERVANCES

Campus policy regarding religious observances requires that faculty make every effort to deal reasonably and fairly with all students who, because of religious obligations, have conflicts with scheduled exams, assignments, or required attendance. For full details, see the campus policy regarding religious observances.

ACCOMMODATION FOR DISABILITIES

If you qualify for accommodations because of a disability, please submit your accommodation letter from Disability Services to your faculty member in a timely manner so that your needs can be addressed. Disability Services determines accommodations based on documented disabilities in the academic environment, but please contact me to discuss how I can help even for conditions not on their list. Information on requesting accommodations is located on the Disability Services website. Contact Disability Services at 303–492–8671 or by email at dsinfo@colorado.edu for further assistance. If you have a temporary medical condition or injury, see the guidelines for Temporary Medical Conditions on the Disability Services website.

I try to provide a positive and supportive learning environment for everyone, and it's always helpful for me to hear what works best for you.

MISCONDUCT, DISCRIMINATION, HARASSMENT, AND/OR RETALIATION

CU Boulder is committed to fostering an inclusive and welcoming learning, working, and living environment. The university will not tolerate acts of sexual misconduct (harassment, exploitation, and assault), intimate partner violence (dating or domestic violence), stalking, or protected-class discrimination or harassment by members of our community. Individuals who believe they have been subject to misconduct or retaliatory actions for reporting a concern should contact the Office of Institutional Equity and Compliance (OIEC) at 303–492–2127 or by email at cureport@colorado.edu. Information about university policies, reporting options, and other support resources can be found on the OIEC website. Please know that faculty and instructors have a responsibility to inform OIEC when made aware of incidents of sexual misconduct, dating and domestic violence, stalking, discrimination, harassment and/or related retaliation, to ensure that individuals impacted receive information about their rights, support resources, and reporting options. To learn more about reporting and support options for a variety of concerns, visit Don't Ignore It.

PREFERRED STUDENT NAMES AND PRONOUNS

CU Boulder recognizes that students' legal information doesn't always align with how they identify. Students may update their preferred names and pronouns via the student portal; those preferred names and pronouns are listed on instructors' class rosters. In the absence of such updates, the name that appears on the class roster is the student's legal name.

CLASSROOM BEHAVIOR

Both students and faculty are responsible for maintaining an appropriate learning environment in all instructional settings, whether in person, remote, or online. Those who fail to adhere to such behavioral standards may be subject to discipline. Professional courtesy and sensitivity are especially important with respect to individuals and topics dealing with race, color, national origin, sex, pregnancy, age, disability, creed, religion, sexual orientation, gender identity, gender expression, veteran status, political affiliation or political philosophy. For more information, see the policies on classroom behavior and the Student Conduct & Conflict Resolution policies.

The policy of the Department of Astrophysical and Planetary Sciences is to ban any use of electronic devices (cellphones, tablets, laptops) in class except as an approved accommodation granted by Disability Services, or as explicitly authorized by the instructor. *In this course* I authorize the use of tablets and laptops for note-taking, but in-person students doing so must do their best to seat themselves with nobody behind them.

REQUIREMENTS FOR COVID–19

As a matter of public health and safety, all members of the CU Boulder community and all visitors to campus must follow university, department, and building requirements and all public health orders in place to reduce the risk of spreading infectious disease. CU Boulder currently requires COVID-19 vaccination and boosters for all faculty, staff and students. Students, faculty and staff must upload proof of vaccination and boosters or file for an exemption based on medical, ethical or moral grounds through the MyCUHealth portal.

The CU Boulder campus is currently mask-optional. However, if public health conditions change and masks are again required in classrooms, students who fail to adhere to masking requirements will be asked to leave class, and students who do not leave class when asked or who refuse to comply with these requirements will be referred to Student Conduct and Conflict Resolution. For more information, see additional policies on classroom behavior and the Student Code of Conduct. If you require accommodation because a disability prevents you from fulfilling these safety measures, please follow the steps in the "Accommodation for Disabilities" statement on this syllabus.

If you feel ill and think you might have COVID-19, if you have tested positive for COVID-19, or if you are unvaccinated or partially vaccinated and have been in close contact with someone who has COVID-19, you should stay home and follow the further guidance of the Public Health Office (contacttracing@colorado.edu). If you are fully vaccinated and have been in close contact with someone who has COVID-19, you do not need to stay home; rather, you should self-monitor for symptoms and follow the further guidance of the Public Health Office (contacttracing@colorado.edu).