A selection of hopefully useful formulae

"The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or for worse, to our pleasure, even though perhaps also to our bafflement, to wide branches of learning."

— Eugene Wigner (1960)

1 Mathematics

1.1 Trigonometric Identities

$$\sin(\alpha \pm \beta) = \sin\alpha\,\cos\beta \pm \cos\alpha\,\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\,\cos\beta \mp \sin\alpha\,\sin\beta$$

$$\sin 2\theta = 2\sin\theta\cos\theta \quad , \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 \quad , \qquad \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Undoing the inverse...

$$\sin\left(\cos^{-1} x\right) = \cos\left(\sin^{-1} x\right) = \sqrt{1-x^2}$$

$$\sin\left(\tan^{-1} x\right) = \frac{x}{\sqrt{1+x^2}} \quad , \qquad \cos\left(\tan^{-1} x\right) = \frac{1}{\sqrt{1+x^2}}$$

$$\tan\left(\sin^{-1} x\right) = \frac{x}{\sqrt{1-x^2}} \quad , \qquad \tan\left(\cos^{-1} x\right) = \frac{\sqrt{1-x^2}}{x}$$

1.2 Vector Identities

Notation: f, g, are scalars; $\mathbf{A}$, $\mathbf{B}$, etc., are vectors.

(1) $\mathbf{A} \cdot \mathbf{B} \times \mathbf{C} = \mathbf{A} \times \mathbf{B} \cdot \mathbf{C} = \mathbf{B} \cdot \mathbf{C} \times \mathbf{A} = \mathbf{B} \times \mathbf{C} \cdot \mathbf{A} = \mathbf{C} \cdot \mathbf{A} \times \mathbf{B} = \mathbf{C} \times \mathbf{A} \cdot \mathbf{B}$

(2) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{C} \times \mathbf{B}) \times \mathbf{A} = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C}$

(3) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) + \mathbf{B} \times (\mathbf{C} \times \mathbf{A}) + \mathbf{C} \times (\mathbf{A} \times \mathbf{B}) = 0$

(4) $(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C})$

(5) $(\mathbf{A} \times \mathbf{B}) \times (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \times \mathbf{B} \cdot \mathbf{D})\mathbf{C} - (\mathbf{A} \times \mathbf{B} \cdot \mathbf{C})\mathbf{D}$

(6) $\nabla(fg) = \nabla(gf) = f\nabla g + g\nabla f$

(7) $\nabla \cdot (f\mathbf{A}) = f\nabla \cdot \mathbf{A} + \mathbf{A} \cdot \nabla f$

(8) $\nabla \times (f\mathbf{A}) = f\nabla \times \mathbf{A} + \nabla f \times \mathbf{A}$

(9) $\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot \nabla \times \mathbf{A} - \mathbf{A} \cdot \nabla \times \mathbf{B}$

(10) $\nabla \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A}) + (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B}$

(11) $\mathbf{A} \times (\nabla \times \mathbf{B}) = (\nabla \mathbf{B}) \cdot \mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B}$

(12) $\nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$

(13) $\nabla^2 f = \nabla \cdot \nabla f$

(14) $\nabla^2 \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla \times \nabla \times \mathbf{A}$

(15) $\nabla \times \nabla f = 0$

(16) $\nabla \cdot \nabla \times \mathbf{A} = 0$

Also, if vectors $\mathbf{A}$ & $\mathbf{B}$ depend on time t, then

$$\frac{\partial}{\partial t}(\mathbf{A} \cdot \mathbf{B}) \;=\; \mathbf{A} \cdot \frac{\partial \mathbf{B}}{\partial t} + \frac{\partial \mathbf{A}}{\partial t} \cdot \mathbf{B} \qquad \frac{\partial}{\partial t}(\mathbf{A} \times \mathbf{B}) \;=\; \mathbf{A} \times \frac{\partial \mathbf{B}}{\partial t} + \frac{\partial \mathbf{A}}{\partial t} \times \mathbf{B}$$

1.3 Coordinate Systems

Conversions between Cartesian and Cylindrical

$$x = r\cos\phi \qquad r = \sqrt{x^2+y^2}$$
$$y = r\sin\phi \qquad \phi = \tan^{-1}(y/x)$$
$$z = z \qquad z = z$$

Conversions between Cartesian and Spherical

$$x = r\cos\phi\sin\theta \qquad r = \sqrt{x^2+y^2+z^2}$$
$$y = r\sin\phi\sin\theta \qquad \theta = \cos^{-1}(z/r)$$
$$z = r\cos\theta \qquad \phi = \tan^{-1}(y/x)$$

Vector & Differential Operators in CARTESIAN Coordinates

$$\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z$$

$$\mathbf{A}\times\mathbf{B} = \mathbf{e}_x(A_yB_z - A_zB_y) + \mathbf{e}_y(A_zB_x - A_xB_z) + \mathbf{e}_z(A_xB_y - A_yB_x)$$

$$\nabla f = \mathbf{e}_x\frac{\partial f}{\partial x} + \mathbf{e}_y\frac{\partial f}{\partial y} + \mathbf{e}_z\frac{\partial f}{\partial z} \quad , \qquad \nabla\cdot\mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\nabla\times\mathbf{F} = \mathbf{e}_x\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) + \mathbf{e}_y\left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right) + \mathbf{e}_z\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$$

$$\text{Volume element} \qquad dV \;=\; dx\,dy\,dz$$

Vector & Differential Operators in CYLINDRICAL Coordinates (r, ϕ, z)

Divergence

$$\nabla\cdot\mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$

Gradient

$$(\nabla f)_r = \frac{\partial f}{\partial r}; \quad (\nabla f)_\phi = \frac{1}{r}\frac{\partial f}{\partial\phi}; \quad (\nabla f)_z = \frac{\partial f}{\partial z}$$

Curl

$$(\nabla\times\mathbf{A})_r = \frac{1}{r}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}$$

$$(\nabla\times\mathbf{A})_\phi = \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}$$

$$(\nabla \times \mathbf{A})_z = \frac{1}{r}\frac{\partial}{\partial r}(rA_\phi) - \frac{1}{r}\frac{\partial A_r}{\partial \phi}$$

Laplacian

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

Laplacian of a vector

$$(\nabla^2 \mathbf{A})_r = \nabla^2 A_r - \frac{2}{r^2}\frac{\partial A_\phi}{\partial \phi} - \frac{A_r}{r^2}$$

$$(\nabla^2 \mathbf{A})_\phi = \nabla^2 A_\phi + \frac{2}{r^2}\frac{\partial A_r}{\partial \phi} - \frac{A_\phi}{r^2}$$

$$(\nabla^2 \mathbf{A})_z = \nabla^2 A_z$$

Components of $(\mathbf{A} \cdot \nabla)\mathbf{B}$

$$(\mathbf{A} \cdot \nabla \mathbf{B})_r = A_r \frac{\partial B_r}{\partial r} + \frac{A_\phi}{r}\frac{\partial B_r}{\partial \phi} + A_z \frac{\partial B_r}{\partial z} - \frac{A_\phi B_\phi}{r}$$

$$(\mathbf{A} \cdot \nabla \mathbf{B})_\phi = A_r \frac{\partial B_\phi}{\partial r} + \frac{A_\phi}{r}\frac{\partial B_\phi}{\partial \phi} + A_z \frac{\partial B_\phi}{\partial z} + \frac{A_\phi B_r}{r}$$

$$(\mathbf{A} \cdot \nabla \mathbf{B})_z = A_r \frac{\partial B_z}{\partial r} + \frac{A_\phi}{r}\frac{\partial B_z}{\partial \phi} + A_z \frac{\partial B_z}{\partial z}$$

Divergence of a tensor

$$(\nabla \cdot \tau)_r = \frac{1}{r}\frac{\partial}{\partial r}(rT_{rr}) + \frac{1}{r}\frac{\partial T_{\phi r}}{\partial \phi} + \frac{\partial T_{zr}}{\partial z} - \frac{T_{\phi\phi}}{r}$$

$$(\nabla \cdot \tau)_\phi = \frac{1}{r}\frac{\partial}{\partial r}(rT_{r\phi}) + \frac{1}{r}\frac{\partial T_{\phi\phi}}{\partial \phi} + \frac{\partial T_{z\phi}}{\partial z} + \frac{T_{\phi r}}{r}$$

$$(\nabla \cdot \tau)_z = \frac{1}{r}\frac{\partial}{\partial r}(rT_{rz}) + \frac{1}{r}\frac{\partial T_{\phi z}}{\partial \phi} + \frac{\partial T_{zz}}{\partial z}$$

Volume element $\quad dV = r\,dr\,d\phi\,dz$

Vector & Differential Operators in SPHERICAL Coordinates (r, θ, ϕ)

Divergence

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

Gradient

$$(\nabla f)_r = \frac{\partial f}{\partial r}; \quad (\nabla f)_\theta = \frac{1}{r}\frac{\partial f}{\partial \theta}; \quad (\nabla f)_\phi = \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}$$

Curl

$$(\nabla \times \mathbf{A})_r = \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\phi) - \frac{1}{r\sin\theta}\frac{\partial A_\theta}{\partial \phi}$$

$$(\nabla \times \mathbf{A})_\theta = \frac{1}{r\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{1}{r}\frac{\partial}{\partial r}(rA_\phi)$$

$$(\nabla \times \mathbf{A})_\phi = \frac{1}{r}\frac{\partial}{\partial r}(rA_\theta) - \frac{1}{r}\frac{\partial A_r}{\partial \theta}$$

Laplacian

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$$

Laplacian of a vector

$$(\nabla^2\mathbf{A})_r = \nabla^2 A_r - \frac{2A_r}{r^2} - \frac{2}{r^2}\frac{\partial A_\theta}{\partial \theta} - \frac{2\cot\theta A_\theta}{r^2} - \frac{2}{r^2\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$(\nabla^2\mathbf{A})_\theta = \nabla^2 A_\theta + \frac{2}{r^2}\frac{\partial A_r}{\partial \theta} - \frac{A_\theta}{r^2\sin^2\theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$(\nabla^2\mathbf{A})_\phi = \nabla^2 A_\phi - \frac{A_\phi}{r^2\sin^2\theta} + \frac{2}{r^2\sin\theta}\frac{\partial A_r}{\partial \phi} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\theta}{\partial \phi}$$

Components of $(\mathbf{A}\cdot\nabla)\mathbf{B}$

$$(\mathbf{A}\cdot\nabla\mathbf{B})_r = A_r\frac{\partial B_r}{\partial r} + \frac{A_\theta}{r}\frac{\partial B_r}{\partial \theta} + \frac{A_\phi}{r\sin\theta}\frac{\partial B_r}{\partial \phi} - \frac{A_\theta B_\theta + A_\phi B_\phi}{r}$$

$$(\mathbf{A}\cdot\nabla\mathbf{B})_\theta = A_r\frac{\partial B_\theta}{\partial r} + \frac{A_\theta}{r}\frac{\partial B_\theta}{\partial \theta} + \frac{A_\phi}{r\sin\theta}\frac{\partial B_\theta}{\partial \phi} + \frac{A_\theta B_r}{r} - \frac{\cot\theta A_\phi B_\phi}{r}$$

$$(\mathbf{A}\cdot\nabla\mathbf{B})_\phi = A_r\frac{\partial B_\phi}{\partial r} + \frac{A_\theta}{r}\frac{\partial B_\phi}{\partial \theta} + \frac{A_\phi}{r\sin\theta}\frac{\partial B_\phi}{\partial \phi} + \frac{A_\phi B_r}{r} + \frac{\cot\theta A_\phi B_\theta}{r}$$

Divergence of a tensor

$$(\nabla\cdot\tau)_r = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 T_{rr}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta T_{\theta r}) + \frac{1}{r\sin\theta}\frac{\partial T_{\phi r}}{\partial \phi} - \frac{T_{\theta\theta} + T_{\phi\phi}}{r}$$

$$(\nabla\cdot\tau)_\theta = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 T_{r\theta}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta T_{\theta\theta}) + \frac{1}{r\sin\theta}\frac{\partial T_{\phi\theta}}{\partial \phi} + \frac{T_{\theta r}}{r} - \frac{\cot\theta T_{\phi\phi}}{r}$$

$$(\nabla\cdot\tau)_\phi = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 T_{r\phi}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta T_{\theta\phi}) + \frac{1}{r\sin\theta}\frac{\partial T_{\phi\phi}}{\partial \phi} + \frac{T_{\phi r}}{r} + \frac{\cot\theta T_{\phi\theta}}{r}$$

Volume element $\qquad dV = r^2 \sin\theta\, dr\, d\theta\, d\phi$

1.4 Special Functions & Series Expansions

Binomial Series

$$\text{For } |x| \ll 1, \qquad (1+x)^\alpha \;=\; 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + \cdots$$

Exponential Functions

$$e^x \;=\; 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

Full-width at half-maximum (FWHM):

$$\text{For} \quad y = \exp\left[-\left(\frac{x}{V_{1/e}}\right)^2\right] \;, \qquad \text{the FWHM} \;=\; V_{1/e} 2\sqrt{\ln 2} \;\approx\; 1.66511 V_{1/e} \;.$$

Dirac Delta Function

$$\int_{-\infty}^{+\infty} dx \; f(x)\,\delta(x-a) \;=\; f(a)$$

$$\delta[g(x)] \;=\; \sum_i \frac{\delta(x-x_i)}{|g'(x_i)|} \qquad \text{where } x_i \text{ are the roots of } g(x).$$

$$\text{One of several limiting forms:} \qquad \delta(x) \;=\; \frac{1}{\pi}\lim_{\epsilon\to 0}\left(\frac{\epsilon}{x^2+\epsilon^2}\right)$$

Error Function

$$\text{erf}(x) \;=\; \frac{2}{\sqrt{\pi}}\int_0^x dt\, e^{-t^2}$$

$$\text{erfc}(x) \;=\; \frac{2}{\sqrt{\pi}}\int_x^\infty dt\, e^{-t^2} \;=\; 1 - \text{erf}(x)$$

For small arguments ($x \ll 1$),

$$\text{erf}(x) \;\approx\; \frac{2}{\sqrt{\pi}}\left(x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \cdots\right)$$

For large arguments ($x \gg 1$),

$$\text{erf}(x) \;\approx\; 1 - \frac{e^{-x^2}}{\sqrt{\pi}}\left(\frac{1}{x} - \frac{1}{2x^3} + \frac{3}{4x^5} - \frac{15}{8x^7} + \cdots\right)$$

1.5 Derivatives and Integrals

Definite Integrals of Gaussians

$$\int_0^\infty dx\, e^{-(x/\sigma)^2} \;=\; \frac{\sigma\sqrt{\pi}}{2}$$

$$\int_0^\infty dx\, x\, e^{-(x/\sigma)^2} \;=\; \frac{\sigma^2}{2}$$

$$\int_0^\infty dx\, x^2\, e^{-(x/\sigma)^2} \;=\; \frac{\sigma^3\sqrt{\pi}}{4}$$

$$\int_0^\infty dx\, x^3\, e^{-(x/\sigma)^2} \;=\; \frac{\sigma^4}{2}$$

$$\int_0^\infty dx\, x^4\, e^{-(x/\sigma)^2} \;=\; \frac{3\sigma^5\sqrt{\pi}}{8}$$

$$\int_0^\infty dx\, x^5\, e^{-(x/\sigma)^2} \;=\; \sigma^6$$

$$\int_0^\infty dx\, x^6\, e^{-(x/\sigma)^2} \;=\; \frac{15\sigma^7\sqrt{\pi}}{16}$$

$$\int_0^\infty dx\, x^n\, e^{-(x/\sigma)^2} \;=\; \frac{\sigma^{n+1}}{2}\Gamma\left(\frac{n+1}{2}\right)$$

1.6 Differential Equations

For a 1st order linear equation of the form

$$\frac{dy}{dx} + P(x)y(x) \;=\; Q(x)$$

use an *Integrating Factor*

$$\mu(x) \;=\; \exp\left[\int^x P(x')\, dx'\right]$$

and the solution is

$$y(x) = \frac{1}{\mu(x)}\left[\int^x Q(x')\mu(x')\, dx' \;+\; C\right] \;.$$

2 Physics

Metric Prefixes

Multiple	Prefix	Symbol	Multiple	Prefix	Symbol
10^{-1}	deci	d	10	deca	da
10^{-2}	centi	c	10^{2}	hecto	h
10^{-3}	milli	m	10^{3}	kilo	k
10^{-6}	micro	μ	10^{6}	mega	M
10^{-9}	nano	n	10^{9}	giga	G
10^{-12}	pico	p	10^{12}	tera	T
10^{-15}	femto	f	10^{15}	peta	P
10^{-18}	atto	a	10^{18}	exa	E
10^{-21}	zepto	z	10^{21}	zetta	Z
10^{-24}	yocto	y	10^{24}	yotta	Y

Physical Constants

Speed of light in vacuum .... $c = 2.99792458 \times 10^{8}$ m s^{-1}
Newton's gravitation constant .. $G = 6.67384 \times 10^{-11}$ N m^2 kg^{-2}
Boltzmann's ideal gas constant . $k_{\rm B} = 1.3806488 \times 10^{-23}$ J K^{-1}
Stefan-Boltzmann constant ..... $\sigma = 5.670373 \times 10^{-8}$ W m^{-2} K^{-4}
Radiation pressure constant .... $a = 4\sigma/c = 7.5657314 \times 10^{-16}$ J m^{-3} K^{-4}
Planck's constant .... $h = 2\pi\hbar = 6.62606957 \times 10^{-34}$ J s
Permittivity of free space .... $\epsilon_0 = 8.8542 \times 10^{-12}$ C^2 N^{-1} m^{-2}
Permeability of free space ...... $\mu_0 = 4\pi \times 10^{-7}$ N s^2 C^{-2}
Energy associated with 1 eV ... $E_{\rm eV} = 1.602176565 \times 10^{-19}$ J

Atomic Constants

Electron charge .... $e = 1.602176565 \times 10^{-19}$ C
Electron mass .... $m_e = 9.10938291 \times 10^{-31}$ kg
Proton mass .... $m_p = 1.672621777 \times 10^{-27}$ kg $\approx$ 1836 m_e
Neutron mass .... $m_n = 1.674927351 \times 10^{-27}$ kg $\approx$ 1839 m_e
Atomic mass unit .... 1 u $= m(^{12}\mathrm{C})/12 = 1.660538921 \times 10^{-27}$ kg
Bohr radius .... $a_0 = \hbar^2/(m_e e^2) = 5.2917721092 \times 10^{-11}$ m
Classical electron radius .... $r_e = e^2/(\hbar c) = 2.8179403267 \times 10^{-15}$ m
Thomson cross section .... $\sigma_{\rm T} = (8\pi/3) r_e^2 = 6.652458734 \times 10^{-29}$ m^2

Astronomical Constants

Solar mass .... $M_\odot = 1.989 \times 10^{30}$ kg
Solar radius .... $R_\odot = 6.963 \times 10^{8}$ m
Solar luminosity .... $L_\odot = 3.83 \times 10^{26}$ J s^{-1}
Solar effective temperature ..... $T_{\rm eff} = 5770$ K
Solar surface gravity .... $g_\odot = 273.79$ m s^{-2} $\log g_\odot$ (cgs) $= 4.4374$
Earth's mass .... $M_\oplus = 5.9736 \times 10^{24}$ kg $\approx$ $3 \times 10^{-6}\, M_\odot$
Earth's radius .... $R_\oplus = 6371$ km
Astronomical unit .... 1 AU $= 1.495978707 \times 10^{11}$ m $\approx$ 215 $R_\odot$
Parsec .... 1 $pc = 3.085677 \times 10^{16}$ m $=$ 3.2616 light years

Unit Conversions

Changing from one set of units to another is enabled by thinking of them as ratios that get multiplied together in a chain. Example:

$$\frac{\textbf{55 miles}}{\textbf{hour}} \times \frac{\text{5280 feet}}{\text{1 mile}} \times \frac{\text{12 inches}}{\text{1 foot}} \times \frac{\text{2.54 cm}}{\text{1 inch}} \times \frac{\text{1 m}}{\text{100 cm}} \times \frac{\text{1 hour}}{\text{3600 s}} = \textbf{24.59 m/s}$$

Significant Figures

You should already know how to count up the number of significant digits in a quantity. In scientific notation, it's usually assumed that every digit given prior to the exponential is significant (e.g., 4.1800×10^7 has five significant digits).

- When combining two quantities (e.g., adding, multiplying), the answer should be given with the *least* number of significant digits from the initial quantities.
- However, when working through multi-step calculations, it's useful to keep at least one more significant digit in the intermediate results than will be needed in the final answer.
- If asked to guess or approximate, the final answer should only have (at most) two significant digits.

Other Sources of Useful Data

NRL Plasma Formulary .... http://wwwppd.nrl.navy.mil/nrlformulary/
Fundamental physical constants from NIST .... http://physics.nist.gov/cuu/Constants/
Zombeck's *Handbook of Space Astronomy & Astrophysics* ... http://ads.harvard.edu/books/hsaa/
Wolfram Alpha (just type a question or equation!) .... http://www.wolframalpha.com/